

> 1.1 Factors, multiples and primes

In this section you will ...

- write a positive integer as a product of prime factors.
- use prime factors to find a highest common factor (HCF) and a lowest common multiple (LCM).

Any **integer** bigger than 1:

- is a prime number, or
- can be written as a product of prime numbers.

Example:

$$46 = 2 \times 23 \quad 47 \text{ is prime} \quad 48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$49 = 7 \times 7 \quad 50 = 2 \times 5 \times 5$$

You can use a **factor tree** to write an integer as a product of its **prime factors**.

This is how to draw a factor tree for 120.

- 1 Write 120.
- 2 Draw branches to two numbers that have a product of 120. Do not use 1 as one of the numbers. Here we have chosen 12 and 10. $120 = 12 \times 10$
- 3 Do the same with 12 and 10. Here $12 = 3 \times 4$ and $10 = 2 \times 5$
- 4 3, 2 and 5 are prime numbers, so circle them.
- 5 Draw two more branches from 4. $4 = 2 \times 2$. Circle the 2s.
- 6 Now all the end numbers are prime, so stop.
- 7 120 is the product of all the end numbers: $120 = 2 \times 2 \times 2 \times 3 \times 5$
- 8 You can check that this is correct using a calculator.

You can also write the result like this: $120 = 2^3 \times 3 \times 5$

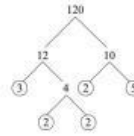
2^3 means $2 \times 2 \times 2$ and the small 3 is an **index**.

Now check that $75 = 3 \times 5^2$

You can use products of prime factors to find the **HCF** and **LCM** of two numbers.

Key words

factor tree
highest common factor (HCF)
index
integer
lowest common multiple (LCM)
prime factor



Worked example 1.1

- a Find the LCM of 120 and 75.
b Find the HCF of 120 and 75.

Answer

- a Write 120 and 75 as products of their prime factors:

$$120 = 2 \times 2 \times 2 \times 3 \times 5$$

$$75 = 3 \times 5 \times 5$$

Look at the prime factors of both numbers.

For the LCM, use the **larger** frequency of each prime factor.

- 120 has three 2s and 75 has no 2s. The LCM must have three 2s.
- 120 has one 3 and 75 has one 3. The LCM must have one 3.
- 120 has one 5 and 75 has two 5s. The LCM must have two 5s.

The LCM is $2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^3 \times 3 \times 5^2 = 8 \times 3 \times 25 = 600$

- b For the HCF use the **smaller** frequency of each factor: there are no 2s in 75, and there is one 3 and one 5 in both numbers.

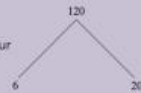
Multiply these factors.

The HCF is $3 \times 5 = 15$

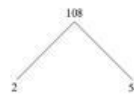
Exercise 1.1**Think like a mathematician**

- 1 The factor tree for 120 in Section 1.1 started with 12×10 .

- a Draw a factor tree for 120 that starts with 6×20 .
b Compare your answer to part a with a partner's. Are your trees the same or different?
c Draw some different factor trees for 120. Can you say how many different trees are possible?
d Do all factor trees for 120 have the same end points?



- 2 a Complete this factor tree for 108.
b Draw a different factor tree for 108.
c Write 108 as a product of its prime factors.
d Compare your factor trees and your product of prime factors with a partner's. Have you drawn the same trees or different ones? Are your trees correct?



- 3 a Draw a factor tree for 200 that starts with 10×20 .
 b Write 200 as a product of prime numbers.
 c Compare your factor tree with a partner's. Have you drawn the same tree or different ones? Are your trees correct?
 d How many different factor trees can you draw for 200 that start with 10×20 ?

- 4 a Draw a factor tree for 330.
 b Write 330 as a product of prime numbers.
 5 Match each number to a product of prime factors.

The first one has been done for you: a and i.

- a 20 ————— i $2^2 \times 5$
 b 24 ————— ii $2 \times 3 \times 7$
 c 42 ————— iii $2^2 \times 3^2 \times 5$
 d 50 ————— iv 2×5^2
 e 180 ————— v $2^2 \times 3$

- 6 Work out the product of each set of prime factors.

- a $3^3 \times 5 \times 7$ b $2^3 \times 5^3$ c $2^2 \times 3^3 \times 11$
 d $2^4 \times 7^2$ e 3×17^2

- 7 Write each of these numbers as a product of prime factors.

- a 28 b 60 c 72
 d 153 e 190 f 275

- 8 a Copy the table and write each number as a product of prime numbers.

Number	Product of prime numbers
35	5×7
70	
140	
280	

- b Add more rows to the table to continue the pattern.
 9 a Write 1001 as a product of prime numbers.
 b Write 4004 as a product of prime numbers.
 c Write 6006 as a product of prime numbers.
 10 a Use a factor tree to write 132 as a product of prime numbers.
 b Write 150 as a product of prime numbers.
 c $132 \times 150 = 19\,800$. Use this fact to write 19 800 as a product of prime numbers.

Tip

You can use a factor tree to help you.

- 11 a Write each of these numbers as a product of prime numbers.
 i 15 ii 15^2 iii 28
 iv 28^2 v 36 vi 36^2
- b What do you notice about your answers to i and ii, iii and iv, v and vi?
- c If $96 = 2^5 \times 3$, show how to find the prime factors of 96^2 . Will your method work for all numbers?
- 12 $40 = 2 \times 2 \times 2 \times 5$ and $28 = 2 \times 2 \times 7$
 Use these facts to find
 a the HCF of 40 and 28 b the LCM of 40 and 28.
- 13 $450 = 2 \times 3 \times 3 \times 5 \times 5$ and $60 = 2 \times 2 \times 3 \times 5$
 Use these facts to find
 a the HCF of 450 and 60 b the LCM of 450 and 60.
- 14 $180 = 2^2 \times 3^2 \times 5$ and $54 = 2 \times 3^3$
 Use these facts to find
 a the HCF of 180 and 54 b the LCM of 180 and 54.
- 15 a Write 45 as a product of prime numbers.
 b Write 75 as a product of prime numbers.
 c Find the LCM of 45 and 75.
 d Find the HCF of 45 and 75.
- 16 a Draw factor trees to find the LCM of 90 and 140.
 b Compare your answer with a partner's. Did you draw the same factor trees? Have you both got the same answer?
- 17 a Write 396 as a product of prime numbers.
 b Write 168 as a product of prime numbers.
 c Find the HCF of 396 and 168.
 d Find the LCM of 396 and 168.
- 18 a Find the HCF of 34 and 58.
 b Find the LCM of 34 and 58.
- 19 Show that the HCF of 63 and 110 is 1.
- 20 37 and 47 are prime numbers.
 a What is the HCF of 37 and 47?
 b What is the LCM of 37 and 47?
 c Write a rule for finding the HCF and LCM of two prime numbers.
 d Compare your answer to part c with a partner's answer. Check your rules by finding the HCF and LCM of 39 and 83.

Tip

Use a calculator to help you.