Worksheet 1

Topic: Solving Logarithmic Equations

Name: ______________________________ Date: ____________ Class: ____________

Key Rules

Equality Property:	log_b(A) = log_b(B) → A = B
Product Rule:	log(A) + log(B) = log(A × B)
Quotient Rule:	log(A) − log(B) = log(A / B)
Exponential Form:	log_b(x) = y means b^y = x

Worked Example A — Equality Property

Solve: $\log_5(3x - 1) = \log_5(x + 7)$

Step 1: Both sides have the same base (base 5). Use the Equality Property:
If log_b(A) = log_b(B) then A = B

Step 2: Set the insides equal: $3x - 1 = x + 7$

Step 3: Solve: $3x - x = 7 + 1 \rightarrow 2x = 8 \rightarrow x = 4$

Step 4: Check: $\log_5(3(4)-1) = \log_5(11)$ ✓ and $\log_5(4+7) = \log_5(11)$ ✓

✓ Final Answer: x = 4

Worked Example B — Quotient Rule

Solve: $\log(6x - 4) - \log 2 = 1$

Step 1: Apply the Quotient Rule: $\log((6x - 4) / 2) = 1$

Step 2: Rewrite in exponential form (base 10): $(3x - 2) = 10^1$ => $3x - 2 = 10$

Step 4: Solve: $3x = 12 \rightarrow x = 4$

✓ Final Answer: x = 4

Questions

Follow the steps shown in the worked examples.

1. Solve: $\log_8(7x - 5) = \log_8(4x + 9)$

Hint: Both sides have the same base — use the Equality Property.

Step 1: Set the insides equal: 7x − 5 = __________

Step 2: Collect x-terms: 7x − ____x = 9 + ____

Step 3: Solve: ____x = ____ → x = ____

Final Answer: x = ____

2. Solve: $\log_{15}(x + 13) = \log_{15}(9x - 5)$

Hint: Same base on both sides — use the Equality Property.

Step 1: Set insides equal: x + 13 = __________

Step 2: Collect x-terms: 13 + 5 = 9x − ____x

Step 3: Solve: ____ = ____x → x = ____

Final Answer: x = ______________________________

3. Solve: log(4x − 10) − log 2 = 1

Hint: Use Quotient Rule first: log(A) − log(B) = log(A / B)

Step 1: Combine logs: log((4x − 10) / ____) = 1

Step 2: Exponential form (base 10): (2x − 5) = 10^____ = ____

Step 3: Solve: 2x = ____ → x = ____

Final Answer: x = ______________________________

Extension:

4. Solve: $\log_6(4x + 7) + \log_6 3 = \log_6 9$

Hint: Use the Product Rule on the left side first, then the Equality Property.

Step 1: Product Rule: $\log_6($ ____ × (4x + 7) $) = \log_6 9$

Step 2: Equality Property: 3(4x + 7) = ____

Step 3: Expand: 12x + ____ = 9

Step 4: Solve: 12x = ____ → x = ____

Final Answer: x = ______________________________

5. Sound levels: L = 10 log R, where L is loudness (decibels) and R is relative intensity. A concert has a loudness of 115 dB. Find R.

Hint: Substitute L = 115 and isolate log R, then convert to exponential form.

Step 1: Substitute: 115 = 10 × log R

Step 2: Divide by 10: log R = ____

Step 3: Exponential form: R = 10^____

Final Answer: R = ______________________________

6. Using the same formula L = 10 log R, a normal conversation has a loudness of 62 dB. Find R for the conversation.

Hint: Same method as Question 5 — substitute L = 62.

Step 1: Substitute: 62 = 10 × log R

Step 2: log R = ____

Step 3: R = 10^____

Final Answer: R = ______________________________

Worksheet 2

Topic: Solving Logarithmic Equations

Name: ____________________________ Date: ____________ Class: ____________

Solve each equation algebraically. Show all steps. Always check for extraneous solutions.

1. Solve: $\log_8(7x - 5) = \log_8(4x + 9)$

2. Solve: $\log_{15}(x + 13) = \log_{15}(9x - 5)$

3. Solve: $\log(4x - 10) - \log 2 = 1$

Extension:

4. Solve: $\log_6(4x + 7) + \log_6 3 = \log_6 9$

5. Solve: $\log_8(x + 5) = 2/3$

Hint: Convert to exponential form: b^y = x. Recall: 8^(2/3) = (³√8)²

6. The loudness of a sound is given by $L = 10 \log R$.

(a) A concert measures 115 dB. Find the relative intensity R.

(b) A normal conversation measures 62 dB. Find the relative intensity R.

7. Using your answers from Question 6, how many times more intense is the concert than the conversation? Show your working clearly.

8. Holly solved $\log_6(x - 5) = 2 - \log_6 x$ and found two solutions: $x = 9$ and $x = -4$.

Explain why one of these solutions is invalid. State the correct answer.

Worksheet 3

Topic: Solving Logarithmic Equations

Name: ____________________________ Date: ____________ Class: ____________

Show complete algebraic working. Justify all steps. Check for extraneous solutions throughout.

1. Solve: $\log_{15}(x + 13) = \log_{15}(9x - 5)$. Verify your answer.

2. Solve: $\log(4x - 10) - \log 2 = 1$. State which logarithm property you used at each step.

3. Solve: $\log_6(4x + 7) + \log_6 3 = \log_6 9$. Explain why you must check your answer.

4. Solve: $\log_8(x + 5) = 2/3$. Show the exponential conversion step explicitly and verify.

5. The loudness formula is $L = 10 \log R$. A concert measures 115 dB; a conversation measures 62 dB.

(a) Find the relative intensity R for each situation.

(b) Calculate exactly how many times more intense the concert is than the conversation.

(c) Express your answer to part (b) in standard form to 3 significant figures.

6. The magnitude of an earthquake is given by $M = (2/3)(\log E - 11.8)$, where E is the energy released in ergs.

The 1906 San Francisco earthquake had an estimated magnitude of 7.9.

(a) Show all algebraic steps to find E.

(b) Express E in standard form.

7. Open-Ended: Write your own logarithmic equation that has logarithmic expressions on both sides, can be solved algebraically without a calculator, and does not produce a quadratic. Solve it and explain why your equation meets all three conditions.