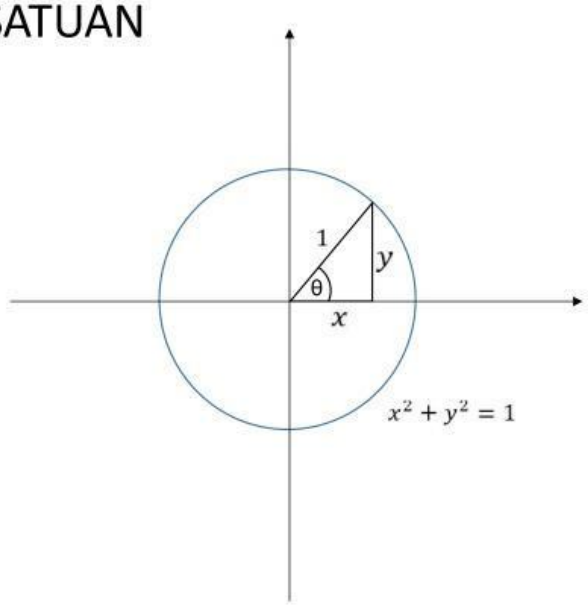
# Identitas Trigonometri

LIVEWORKSHEETS

# PERBANDINGAN TRIGONOMETRI DENGAN MENGGUNAKAN LINGAKARAN SATUAN

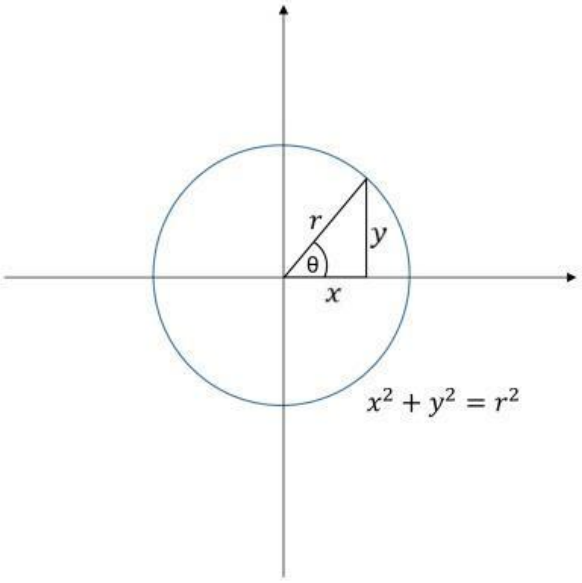
- $sin\ \theta = \frac{x}{1}$
- $cos\ \theta = \frac{y}{1}$
- $tan\ \theta = \frac{x}{y}$
- $sec\ \theta = \frac{1}{y}$
- $csc\ \theta = \frac{1}{x}$
- $cot\ \theta = \frac{y}{x}$




LIVEWORKSHEETS

# PERBANDINGAN TRIGONOMETRI DENGAN JARI-JARI r

- $sin\ \theta = \frac{x}{r}$
- $cos\ \theta = \frac{y}{r}$
- $tan\ \theta = \frac{x}{y}$
- $sec\ \theta = \frac{r}{y}$
- $csc\ \theta = \frac{r}{x}$
- $cot\ \theta = \frac{y}{x}$




LIVEWORKSHEETS

# Identitas Kebalikan

1. $sin\,\theta = \frac{1}{csc\,\theta}$ atau $csc\,\theta = \frac{1}{sin\,\theta}$
2. $cos\,\theta = \frac{1}{sec\,\theta}$ atau $sec\,\theta = \frac{1}{cos\,\theta}$
3. $tan\,\theta = \frac{1}{cot\,\theta}$ $atau\, cot\,\theta = \frac{1}{tan\,\theta}$ (Silahkan ananda buktikan!!)

Bukti:

1. $sin\,\theta = \frac{x}{r}$ atau $csc\,\theta = \frac{r}{x}$

$= \frac{1}{\frac{r}{x}}$ $= \frac{1}{\frac{x}{r}}$

$= \frac{1}{csc\,\theta}$ $= \frac{1}{sin\,\theta}$

Terbukti bahwa $sin\,\theta = \frac{1}{csc\,\theta}$

Terbukti bahwa $csc\,\theta = \frac{1}{sin\,\theta}$

2. $cos\,\theta = \frac{y}{r}$ atau $sec\,\theta = \frac{r}{y}$

$= \frac{1}{\frac{r}{y}}$ $= \frac{1}{\frac{y}{r}}$

$= \frac{1}{sec\,\theta}$ $= \frac{1}{cos\,\theta}$

Terbukti bahwa $cos\,\theta = \frac{1}{sec\,\theta}$

Terbukti bahwa $sec\,\theta = \frac{1}{cos\,\theta}$

LIVEWORKSHEETS

# Identitas Kebalikan

1. $sin\,\theta = \frac{1}{csc\,\theta}$ atau $csc\,\theta = \frac{1}{sin\,\theta}$
2. $cos\,\theta = \frac{1}{sec\,\theta}$ atau $sec\,\theta = \frac{1}{cos\,\theta}$
3. $tan\,\theta = \frac{1}{cot\,\theta}$ $atau\, cot\,\theta = \frac{1}{tan\,\theta}$ (Silahkan ananda buktikan!!)

Bukti:

Cara I:

1. $sin\,\theta = \frac{x}{r}$ atau $csc\,\theta = \frac{r}{x}$

$= \frac{1}{\frac{r}{x}}$ $= \frac{1}{\frac{x}{r}}$

$= \frac{1}{csc\,\theta}$ $= \frac{1}{sin\,\theta}$

Cara II:

$\frac{1}{csc\,\theta} = \frac{1}{\frac{r}{x}}$ atau $\frac{1}{sin\,\theta} = \frac{1}{\frac{x}{r}}$

$= \frac{x}{r}$ $= \frac{r}{x}$

$= sin\,\theta$ $= csc\,\theta$

Dengan Cara I dan Cara II diperoleh bahwa

$sin\,\theta = \frac{1}{csc\,\theta}$ dan $csc\,\theta = \frac{1}{sin\,\theta}$ **terbukti**

LIVEWORKSHEETS

# Identitas Perbandingan

1. $tan\ \theta = \frac{sin\ \theta}{cos\ \theta}$
2. $cot\ \theta = \frac{cos\ \theta}{sin\ \theta}$ (Silahkan ananda buktikan!!)

Bukti :

1. CARA I: $\frac{sin\ \theta}{cos\ \theta} = \frac{\frac{x}{r}}{\frac{y}{r}}$

$= \frac{x}{y} = tan\ \theta$

CARA II: $tan\ \theta = \frac{x}{y}$

$= \frac{\frac{x}{r}}{\frac{y}{r}} = \frac{sin\ \theta}{cos\ \theta}$

Dari cara I dan cara II diperoleh bahwa $tan\ \theta = \frac{sin\ \theta}{cos\ \theta}$ **terbukti**

LIVEWORKSHEETS

# Identitas Phytagoras

Berdasarkan teorema phytagoras

$x^2 + y^2 = r^2$

Kita bagi kedua ruas dengan $r^2$, sehingga

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

Diketahui $sin\ \theta = \frac{x}{r}$ dan $cos\ \theta = \frac{y}{r}$, sehingga

$(sin\ \theta)^2 + (cos\ \theta)^2 = 1$ ..... (i)

Bentuk alternatif:

$(sin\ \theta)^2 = 1 - (cos\ \theta)^2$

$(cos\ \theta)^2 = 1 - (sin\ \theta)^2$

Berdasarkan identitas phytagoras (i)

$(sin\ \theta)^2 + (cos\ \theta)^2 = 1$

Dapat diperoleh identitas kedua dengan cara membagi kedua ruas dengan $(sin\ \theta)^2$ yaitu

$$\frac{(sin\ \theta)^2}{(sin\ \theta)^2} + \frac{(cos\ \theta)^2}{(sin\ \theta)^2} = \frac{1}{(sin\ \theta)^2}$$

$$1 + \frac{(cos\ \theta)^2}{(sin\ \theta)^2} = \frac{1}{(sin\ \theta)^2}$$

Diketahui $cot\ \theta = \frac{cos\ \theta}{sin\ \theta}$ dan $csc\ \theta = \frac{1}{sin\ \theta}$, sehingga

$1 + (cot\ \theta)^2 = (csc\ \theta)^2$ ..... (ii)

Bentuk alternatif:

$(cot\ \theta)^2 = (csc\ \theta)^2 - 1$

$1 = (csc\ \theta)^2 - (cos\ \theta)^2$

LIVEWORKSHEETS

Tugas

1. Buktikan identitas kebalikan berikut
   a. $tan\ \theta = \frac{1}{cot\ \theta}$
   b. $cot\ \theta = \frac{1}{tan\ \theta}$
2. Buktikan identitas perbandingan dari $cot\ \theta = \frac{cos\ \theta}{sin\ \theta}$
3. Sederhanakanlah
   a. $-\frac{csc\ \theta}{cot\ \theta}$
   b. $(1 - (sin\ \theta)^2)(1 + (cot\ \theta)^2)$
4. Buktikan identitas berikut $sin\ \theta\ sec\ \theta = tan\ \theta$

LIVEWORKSHEETS