رياضيات 1

Mathematics 1

The Functions

اعداد

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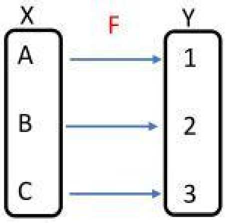
LIVEWORKSHEETS

* Defintion :

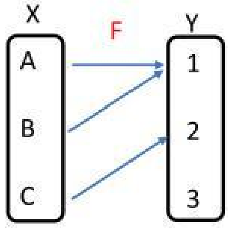
A functions is a relation between the elements of two unempty sets, where every element in a set has only one image in the other set.

this can be written in the form $y = f(x)$ *or* $f : X \to Y$

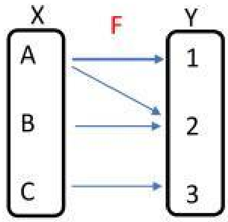
x is independent variable, y is dependen variable.



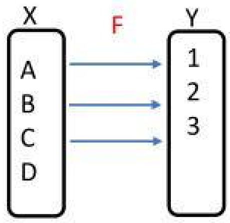
Function



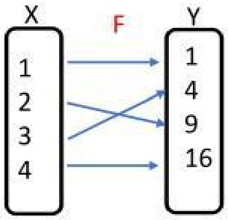
Function



Not Function



Not Function



$f : X \to Y \quad , \quad f(x) = y$

$1 \to 1 \qquad f(1) = 1$

$2 \to 9 \qquad f(2) = 9$

$3 \to 4 \qquad f(3) = 4$

$4 \to 16 \qquad f(4) = 16$

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*Domain of the function :

where X is the domain of the function f and may be denoted by D_f .

Y is the Co-domain of f.

*Range of the function :

the range is a set of all image of the elements of the domain and it is denoted by R_f .

we note that $R_f \subset Y$

*Remarks to Find the Nutral Domain :

1.For rational functions the domain is R except any number make the dominator to be zero i.e. domain =R- {zeros of dominator}.

2.When the function contain square root or any even root $\sqrt[4]{\ }$, $\sqrt[6]{\ }$ ....*hence the domain will be R* except the numbers which make the value under the root sign to be negative value.

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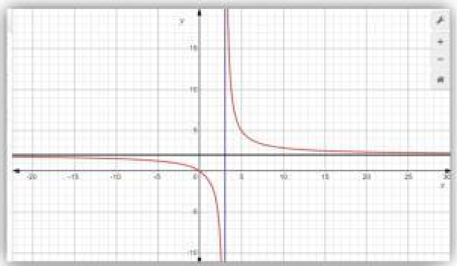
Example : Find the domain and range of the functions

1) $f(x) = \dfrac{2x}{x-3}$ R- {zeros of dominator}

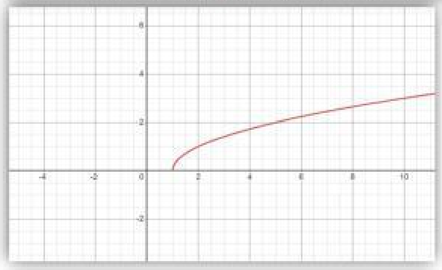


$x-3=0 \;\rightarrow\; x=3 \rightarrow D_f = R-\{3\}$

$y = \dfrac{2x}{x-3} \rightarrow yx-3y=2x \rightarrow \quad yx-2x=3y \quad \rightarrow x(y-2)=3y \rightarrow x = \dfrac{2y}{y-2}$

$R_f = R-\{2\}$

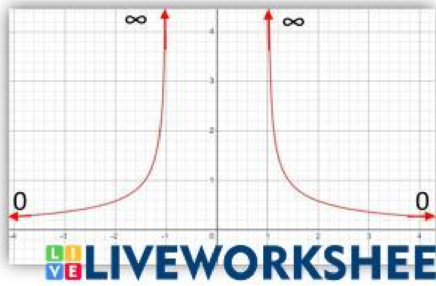
2) $f(x) = \sqrt{x-1}$ the value under the root ≥ 0



$x-1 \geq 0 \;\rightarrow\; x \geq 1 \;\rightarrow\; D_f = [1,\infty[\quad or \quad [1,\infty)$

$R_f = [0,\infty[$

3) $y = \dfrac{1}{\sqrt{x^2-1}}$ the value under the root > 0



$x^2-1>0 \rightarrow x^2>1 \rightarrow \pm x>1$

$x>1 \quad or \quad -x>1$

$x>1 \quad or \quad x<-1$

$D_f = R-[-1,1]$

$at\, x=\pm 1 \;\rightarrow\; y=\infty \qquad , \qquad x=\pm\infty \;\rightarrow\; y=0$

$R_f =]0,\infty[$

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4) $f(x)=\sqrt{4-\sqrt{x}}$

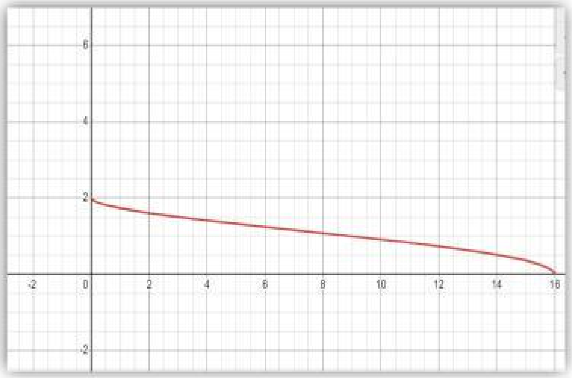
$\sqrt{x} \quad \rightarrow \boxed{x \geq 0}$

$\sqrt{4-\sqrt{x}} \rightarrow 4-\sqrt{x} \geq 0 \rightarrow 4 \geq \sqrt{x} \rightarrow \boxed{16 \geq x}$

$0 \leq x \leq 16 \quad , \quad D_f=[0,16]$

$at\ x=0 \rightarrow y=2 \quad , \quad x=16 \rightarrow y=0$

$R_f=[0,2]$



5) $y=\sqrt{4-x^2}$

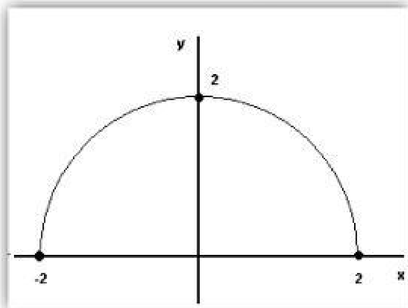
$4-x^2 \geq 0 \rightarrow 4 \geq x^2 \rightarrow 2 \geq \pm x$

$2 \geq x \quad or \quad 2 \geq -x$

$-2 \leq x$

$-2 \leq x \leq 2 \Rightarrow D_f=[-2,2]$

$R_f=[0,2]$



*) $y=\sqrt{4-x^2} \Rightarrow y$ is always positive

$y^2=4-x^2 \quad \rightarrow \quad x^2+y^2=4$

$\boxed{x^2+y^2=a^2}$

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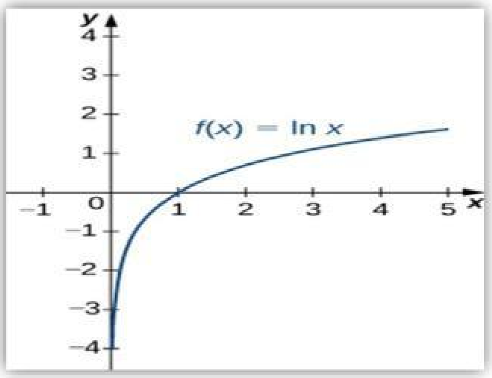
*) $y = \ln x$ $\boxed{\text{the value inside } \ln > 0}$

$x > 0 \quad \rightarrow \quad D_f =]0, \infty[$

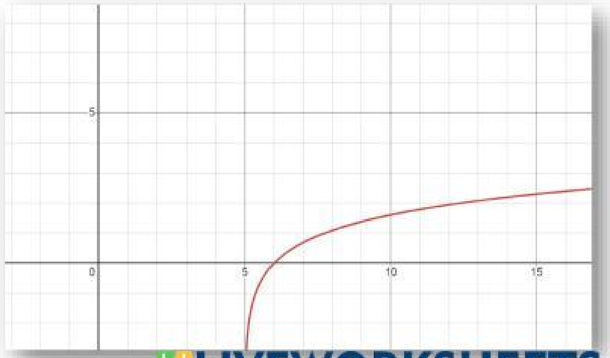
$R_f =]-\infty, \infty[= R$



4) $y = \ln(x-5)$

$x - 5 > 0 \quad \rightarrow \quad x > 5 \quad \rightarrow \quad D_f =]5, \infty[$

$R_f =]-\infty, \infty[\; = R$



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Example: Find the range of the functions

1) $y = \dfrac{1}{2 - \sin x}$

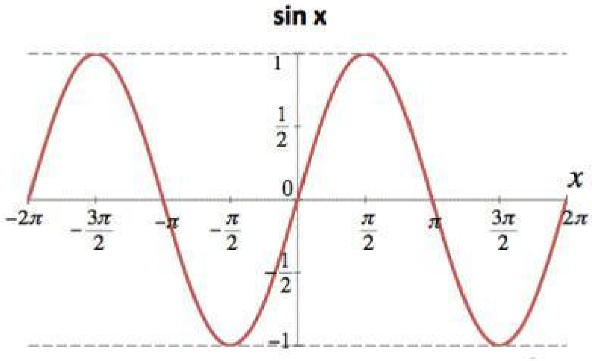
$$2y - y\sin x = 1 \;\rightarrow\; y\sin x = 2y - 1 \;\rightarrow\; \sin x = \frac{2y-1}{y}$$

$$\sin x = 2 - \frac{1}{y}$$

$$\boxed{-1 \le \sin x \le 1}$$

$$-1 \le 2 - \frac{1}{y} \le 1 \;\rightarrow\; -3 \le \frac{-1}{y} \le -1 \;\rightarrow\; 3 \ge \frac{1}{y} \ge 1$$

$$\frac{1}{3} \le y \le 1 \qquad \Rightarrow \mathrm{R}_f = [\frac{1}{3}, 1]$$



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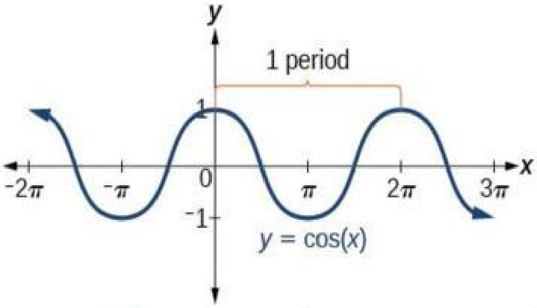
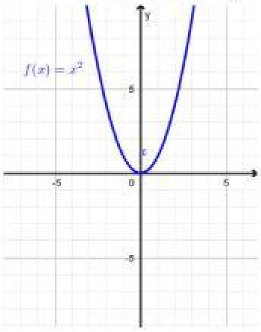
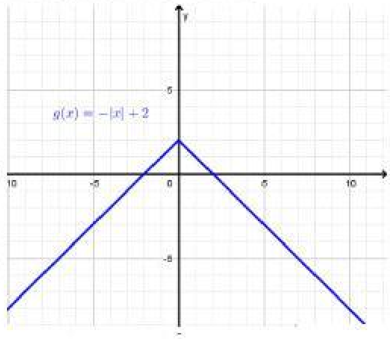
Even and Odd Functions

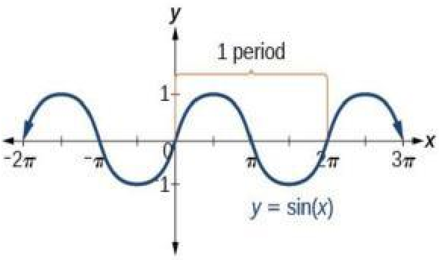
$f(x)$ is an even function if $\boxed{f(-x) = f(x)}$ and has Y-axis as a symmetric line of it

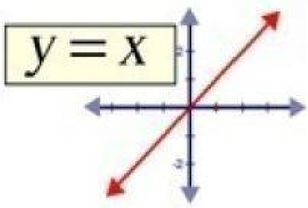


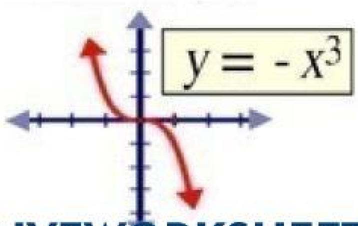




$f(x)$ is an odd function if $\boxed{f(-x) = -f(x)}$ and the curve of $f(x)$ *is* symmetric about the origin







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Ex : Check the following functions even, odd, or not : -

1) $f(x) = x^2 - 1$

$f(-x) = (-x)^2 - 1 = x^2 - 1 = f(x)$

$\therefore\ f(x)$ *is even function*

2) $f(x) = x^2 + x$

$f(-x) = (-x)^2 + (-x) = x^2 - x \neq f(x)$

and $f(-x) \neq -f(x)$

$\therefore\ f(x)$ *neither odd nor even function*

3) $f(x) = \dfrac{x^3 + 2x}{x^4 - 3x^2 + 6}$

$$f(-x) = \frac{(-x)^3 + 2(-x)}{(-x)^4 - 3(-x)^2 + 6} = \frac{-(x^3 + 2x)}{x^4 - 3x^2 + 6} = -f(x)$$

$\rightarrow\ f(-x) = -f(x)$

$\therefore\ f(x)$ *is odd function*

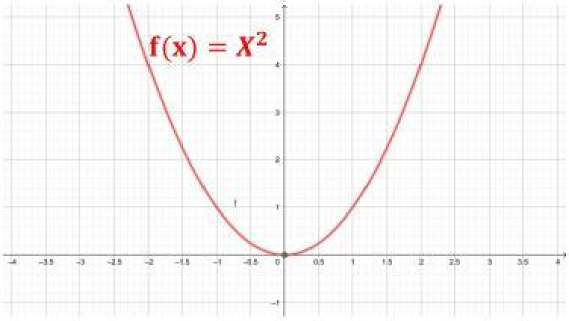
LIVEWORKSHEETS

Types of the function

1. One to one function (injective)

$\boxed{\forall x \in D_f\ ,\ x_1 \neq x_2 \ \Rightarrow f(x_1) \neq f(x_2)} \ \Rightarrow \ \therefore f(x_1) = f(x_2) \Rightarrow \ x_1 = x_2$

$Ex:\ f(x) = x^2$



$let\ \ f(x_1) = f(x_2)$

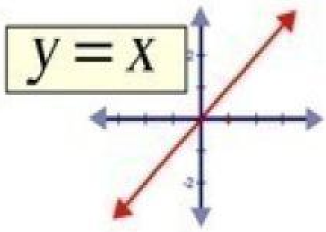
$x^2{}_1 = x^2{}_2 \ \Rightarrow \ x^2{}_1 - x^2{}_2 = 0$

$(x_1 - x_2)(x_1 + x_2) = 0$

$x_1 = x_2 \quad or \qquad x_1 = -x_2$

$f(x)$ is not one to one function

$Ex:\ f(x) = x$



$let\ \ f(x_1) = f(x_2)$

$x_1 = x_2$

$f(x)$ is one to one function

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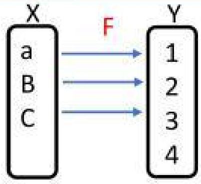
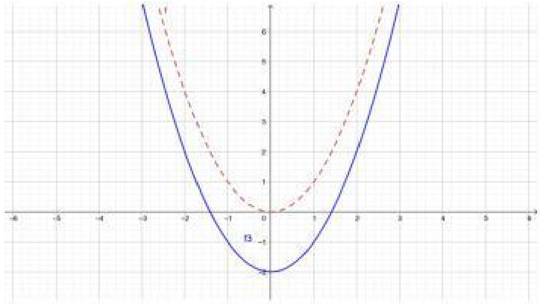
2. Onto function (surjective)

Range = Co_domain $(R_f = Y)$



(Co_domain) $Y = \{1,2,3,4\}$, (Range) $R_f = \{1,2,3\} \Rightarrow \because Y \neq R_f$

$\therefore f(x)$ is not Onto function



Ex: Is $f(x) = x^2 - 2$ Onto where $f: R \to R$?

$(f: domain \to Co\_domain)$

$R_f = [-2, \infty[$, $Y = R$ $\to R_f \neq Y$ $\to \therefore f(x)$ is not onto function

Ex: Is $f(x) = x^2 - 2$ Onto where $f: R \to [-2, \infty[$?

$R_f = [-2, \infty[$, $Y = [-2, \infty[$ $\to R_f = Y$ $\to \therefore f(x)$ is onto function

3. Bijective function

if it is one to one (injective) and onto (surjective)

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*Invers of a Function:

A function $f(x)$ has an invers function $f^{-1}(x)$ if and only if it is bijective (or at least is one to one)

Note that :

1. $f^{-1}(x) \neq \dfrac{1}{f(x)}$

2. The domain of $f^{-1}(x)$ is the co_domain of $f(x)$ and its range is the domain of f

3. The function and its invers are symmetric at $y = x$

Ex: Show that $f(x) = 2x + 1$ has invers and find it

$D_f = R \quad , \quad R_f = R$

let $f(x_1) = f(x_2)$

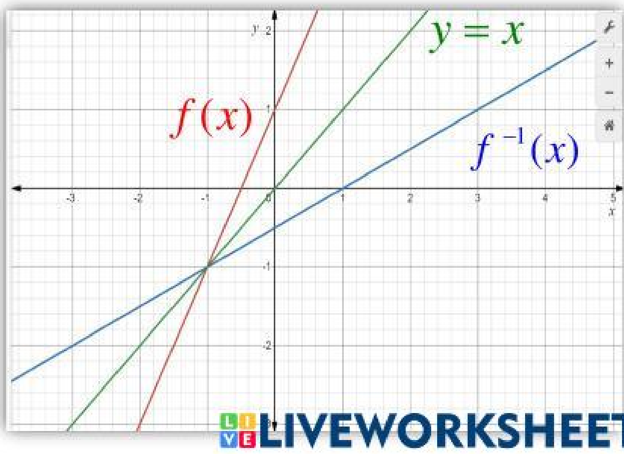
$2x_1 + 1 = 2x_2 + 1 \Rightarrow 2x_1 - 2x_2 = 0 \Rightarrow 2(x_1 - x_2) = 0$

$\therefore\ x_1 = x_2 \Rightarrow f(x)$ is one to one function $\Rightarrow f(x)$ has invers

$\boxed{f(x) = y = 2x + 1} \Rightarrow x = \dfrac{y-1}{2}$

$f^{-1}(y) = \dfrac{y-1}{2}$

$\boxed{f^{-1}(x) = \dfrac{x-1}{2}}$



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Let f , g two functions
$f(g(x)) = x$ and $g(f(x)) = x$
$\therefore$ g is the invers of f and f is the invers of g

$f(x) = 2x+1$, $f^{-1}(x) = \frac{x-1}{2} = g(x)$

$f(g(x)) = 2(\frac{x-1}{2}) + 1 = x$ and $g(f(x)) = \frac{(2x+1)-1}{2} = x$

$\therefore$ g is the invers of f and f is the invers of g

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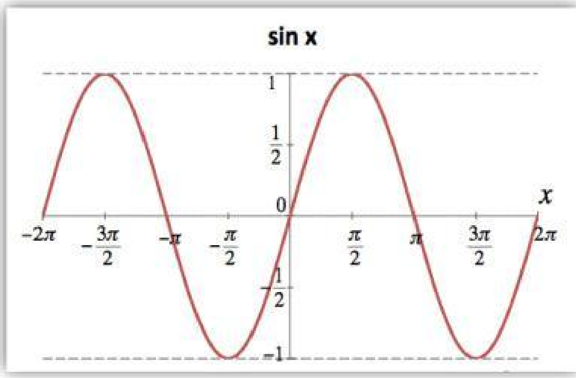
Properties of Function

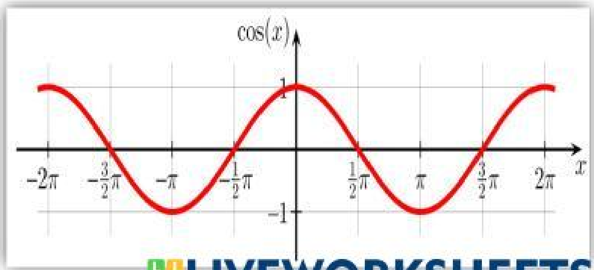
* **Periodic Function**

$$\boxed{f(x \pm p) = f(x)}$$



$$Ex: \; f(x) = \sin(x)$$

$$p = 2\pi \;\; \rightarrow \;\; \sin(x \pm 2\pi) = \sin(x)$$



$$Ex: \; f(x) = \cos(x)$$

$$p = 2\pi \;\; \rightarrow \;\; \cos(x \pm 2\pi) = \cos(x)$$

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Thank you

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