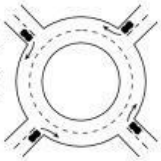
24. A gardener wants to plant twenty trees (maples and lindens) along an avenue in the park. The number of trees between any two maples must not be equal to three. Of these twenty trees, what is the greatest number of maples that the gardener can plant?
A) 8 **B)** 10 **C)** 12 **D)** 14 **E)** 16

25. Andrew and Daniel recently took part in a race. After they had finished, they noticed that Andrew finished ahead of twice as many runners as finished ahead of Daniel, and that Daniel finished ahead of 1.5 times as many runners as finished ahead of Andrew. Andrew finished in 21st place. How many runners took part in the race?
A) 31 **B)** 41 **C)** 51 **D)** 61 **E)** 81

26. Four cars enter a roundabout at the same time, each one from a different direction, as shown in the diagram. Each of the cars drives less than once round the roundabout, and no two cars leave the roundabout in the same direction. How many different ways are there for the cars to leave the roundabout?

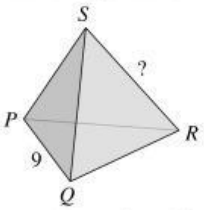


A) 9 **B)** 12 **C)** 15 **D)** 24 **E)** 81

27. A sequence starts $1, -1, -1, 1, -1, -1, 1$. After the seventh term, every term is equal to the product of the two preceding terms. For example, the sixth term is equal to the product of the fourth term and the fifth term. What is the sum of the first 2013 terms?
A) -1006 **B)** -671 **C)** 0 **D)** 671 **E)** 1007

28. Ria bakes six raspberry pies one after the other, numbering them 1 to 6 in order, with the first being number 1. Whilst she is doing this, her children sometimes run into the kitchen and eat the hottest pie. Which of the following could not be the order in which the pies are eaten?
A) $1, 2, 3, 4, 5, 6$ **B)** $1, 2, 5, 4, 3, 6$ **C)** $3, 2, 5, 4, 6, 1$ **D)** $4, 5, 6, 2, 3, 1$ **E)** $6, 5, 4, 3, 2, 1$

29. Each of the four vertices and six edges of a tetrahedron is marked with one of the ten numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 11 (number 10 is omitted). Each number is used exactly once. For any two vertices of the tetrahedron, the sum of two numbers at these vertices is equal to the number on the edge connecting these two vertices. The edge PQ is marked with the number 9. Which number is used to mark edge RS?



A) 4 **B)** 5 **C)** 6 **D)** 8 **E)** 11

30. A positive integer N is smaller than the sum of its three greatest divisors (naturally, excluding N itself). Which of the following statements is true?
A) All such N are divisible by 4. **B)** All such N are divisible by 5.
C) All such N are divisible by 6. **D)** All such N are divisible by 7.
E) There is no such N.

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KANGAROO 2013

C

Time allowed: 75 min
Calculators are not permitted

Cadet
7–8 grades

Questions for 3 points

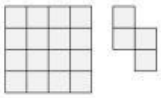
1. In the picture, the big triangle is equilateral and has area 9. The lines are parallel to the sides and divide the sides into three equal parts. What is the area of the shaded part?

A) 1 **B)** 4 **C)** 5 **D)** 6 **E)** 7

2. It is true that $\frac{1111}{101} = 11$. What is the value of $\frac{3333}{101} + \frac{6666}{303}$?
A) 5 **B)** 9 **C)** 11 **D)** 55 **E)** 99

3. The difference between the largest 2-digit positive integer divisible by 7 and the smallest 2-digit positive integer divisible by 7 is equal to:
A) 70 **B)** 77 **C)** 84 **D)** 91 **E)** 98

4. Ann has the square sheet of paper shown on the left diagram (both sides of the sheet are lined). By cutting along the lines of the square, she cuts out copies of the shape shown on the right diagram. What is the smallest possible number of cells remaining?



A) 0 **B)** 2 **C)** 4 **D)** 6 **E)** 8

5. Roo wants to tell Kanga a number with the product of its digits equal to 24. What is the sum of the digits of the smallest number that Roo could tell Kanga?
A) 6 **B)** 8 **C)** 9 **D)** 10 **E)** 11

6. A bag contains balls of four different colours. Two are red, three are blue, four are green and five are black. Balls are taken from the bag without looking, and not returned. What is the smallest number of balls that should be taken from the bag to be sure that two balls of the same colour have been taken?
A) 2 **B)** 4 **C)** 5 **D)** 8 **E)** 14

7. Alex lights a candle every ten minutes. Each candle burns for 40 minutes and then goes out. How many candles are alight 55 minutes after Alex lit the first candle?
A) 2 **B)** 3 **C)** 4 **D)** 5

LIVEWORKSHEETS

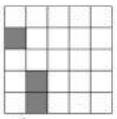
8. The average number of children in five families cannot be:
A) 0.2 **B)** 1.2 **C)** 2.2 **D)** 2.4 **E)** 2.5

9. Mark and Liza stand on opposite sides of a circular fountain. They then start to run clockwise round the fountain. Mark's speed is $\frac{9}{8}$ of Liza's speed. How many circuits has Liza completed when Mark catches up with her for the first time?
A) 4 **B)** 8 **C)** 9 **D)** 2 **E)** 72

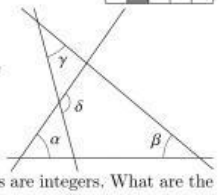
10. The positive integers x, y and z satisfy $x \cdot y = 14$, $y \cdot z = 10$ and $z \cdot x = 35$. What is the value of $x + y + z$?
A) 10 **B)** 12 **C)** 14 **D)** 16 **E)** 18

Questions for 4 points

11. Carina and a friend are playing a game of "battleships" on a 5×5 board. Carina has already placed two ships as shown. She still has to place a 3×1 ship so that it covers exactly three cells. No two ships can have a point in common. How many positions are there for her 3×1 ship?
A) 4 **B)** 5 **C)** 6 **D)** 7 **E)** 8

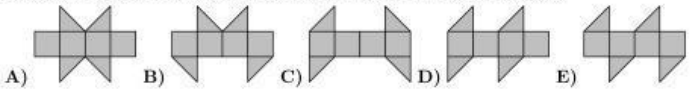


12. In the diagram, $\alpha = 55°$, $\beta = 40°$ and $\gamma = 35°$. What is the value of δ?
A) 100° **B)** 105° **C)** 120° **D)** 125° **E)** 130°



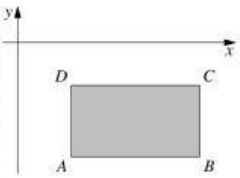
13. The perimeter of a trapezium is 5 and the lengths of its sides are integers. What are the smallest two angles of the trapezium?
A) 30° and 30° **B)** 60° and 60° **C)** 45° and 45° **D)** 30° and 60° **E)** 45° and 90°

14. One of the following nets cannot be folded to form a cube. Which one?



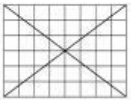
15. Vasya wrote down several consecutive integers. Which of the following could not be the percentage of odd numbers among them?
A) 40 **B)** 45 **C)** 48 **D)** 50 **E)** 60

16. The edges of rectangle $ABCD$ are parallel to the coordinate axes. $ABCD$ lies below the x-axis and to the right of the y-axis, as shown in the figure. The coordinates of the four points A, B, C and D are all integers. For each of these points we calculate the value y-coordinate $\div$ x-coordinate. Which of the four points gives the least value?
A) A **B)** B **C)** C **D)** D **E)** It depends on the rectangle



17. All 4-digit positive integers with the same four digits as in the number 2013 are written on the blackboard in an increasing order. What is the largest possible difference between two neighbouring numbers on the blackboard?
A) 702 **B)** 703 **C)** 693 **D)** 793 **E)** 198

18. In the 6×8 grid shown, 24 of the cells are not intersected by either diagonal. When the diagonals of a 6×10 grid are drawn, how many of the cells are not intersected by either diagonal?
A) 28 **B)** 29 **C)** 30 **D)** 31 **E)** 32

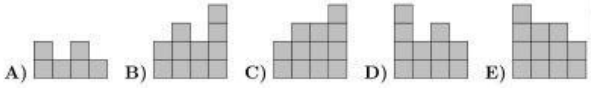


19. Andy, Betty, Cathie, Dannie and Eddy were born on 20/02/2001, 12/03/2000, 20/03/2001, 12/04/2000 and 23/04/2001 (day/month/year). Andy and Eddy were born in the same month. Also, Betty and Cathie were born in the same month. Andy and Cathie were born on the same day of different months. Also, Dannie and Eddy were born on the same day of different months. Which of these children is the youngest?
A) Andy **B)** Betty **C)** Cathie **D)** Dannie **E)** Eddy

20. John has made a building of cubes. In the picture this building and John standing besides it are shown from above. In each cell you see the number of cubes in that particular tower. Which shape will John see?

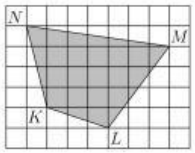
John

4	2	3	2
3	3	1	2
2	1	3	1
1	2	1	2



Questions for 5 points

21. The diagram shows a shaded quadrilateral $KLMN$ drawn on a grid. Each cell of the grid has sides of length 2 cm. What is the area of $KLMN$?
A) 96 cm^2 **B)** 84 cm^2 **C)** 76 cm^2 **D)** 88 cm^2 **E)** 104 cm^2



22. Let S be the number of squares among the integers from 1 to 2013^6. Let Q be the number of cubes among the same integers. Then:
A) $S = Q$ **B)** $2S = 3Q$ **C)** $3S = 2Q$ **D)** $S = 2013Q$ **E)** $S^3 = Q^2$

23. John chooses a 5-digit positive integer and deletes one of its digits to make a 4-digit number. The sum of this 4-digit number and the original 5-digit number is 52713. What is the sum of the digits of the original 5-digit number?
A) 26 **B)** 20 **C)** 23 **D)** 1[illegible] **E)** [illegible]

LIVEWORKSHEETS