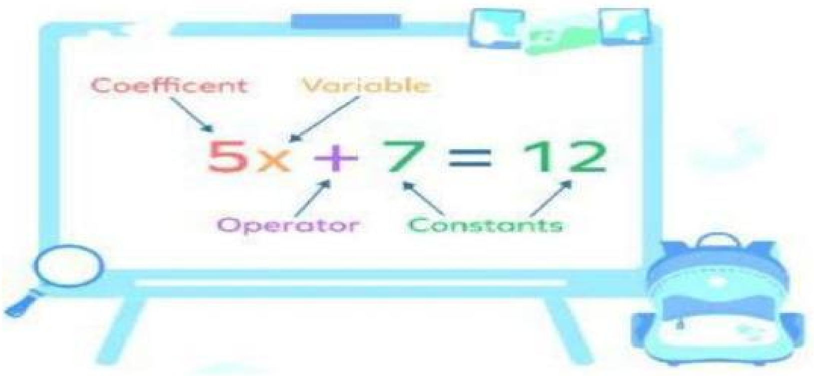
LEARN ENABLED SUMMER BOOT CAMP

MATHEMATICS FOR JS2 & 3

ALGEBRAIC EQUATION

BASIC ALGEBRA

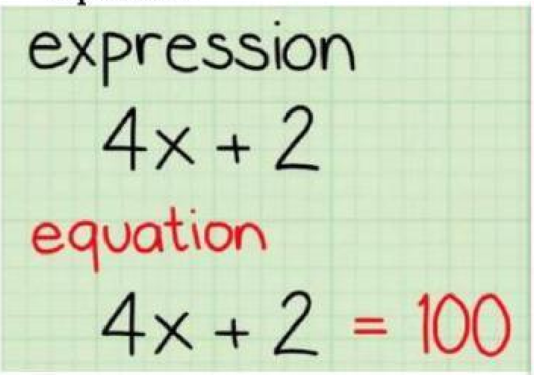
- This topic cover simplifies basic algebra (solve algebraic expressions, factorization
- and expansion of algebraic equations and solving algebraic fractions), show algebraic
- equation as unknown to be subject formula and solving the simultaneous linear
- equation with two variables.



INTRODUCTION TO BASIC ALGEBRA :HOW TO SOLVE ALGEBRAIC EXPRESSIONS

PART 1 : UNDERSTANDING THE BASIC

Understand the difference between an algebraic expression and algebraic equation.



a. Algebraic Expression : a mathematical phrase that contain number/variables. It ***<u>does not contain an equal sign and cannot be solved</u>***

b. Algebraic Equation : ***<u>can be solved</u>*** and does include a series of algebraic

PART 2 : KNOW HOW TO COMBINE LIKE TERMS

Combining like terms just mean adding up (or subtracting) the terms of the same degree. This mean x^2 combined with x^2, x^3 combined with x^3 and numbers (such as 8 or 5) can be combined.

$$=3x^2+5+4x^3-x^2+2x^3+9$$

$$=3x^2-x^2+4x^3+2x^3+5+9$$

$$=2x^2+6x^3+14$$

INTRODUCTION TO BASIC ALGEBRA :HOW TO SOLVE ALGEBRAIC EXPRESSIONS

PART 3 : KNOW HOW TO FACTOR A NUMBER

$$3x + 15 = 9x + 30$$
$$\frac{3x + 15}{3} = \frac{9x + 30}{3}$$
$$\frac{3x}{3} + \frac{15}{3} = \frac{9x}{3} + \frac{30}{3}$$
$$x + 5 = 3x + 10$$

With an algebraic equation, you can simplify it by factoring out a common term.
Look at the coefficients of all terms. If there is a number that can "factor out" by dividing each term by that number......then you have simplified .

PART 4 : HOW TO ISOLATE A VARIABLE

An algebraic equation, your goal is to get the variable, often known as x, on one side of the equation, while placing the constant terms on the other side of the equation.
x can isolate by division, multiplication, addition, subtraction or other operation.

$$5x + 15 = 65$$
$$\frac{5x}{5} + \frac{15}{5} = \frac{65}{5}$$
$$x + 3 = 13$$
$$x + 3 - 3 = 13 - 3$$
$$\boxed{x = 10}$$

SOLVE ALGEBRAIC EXPRESSIONS

EXAMPLE: Solve the algebraic expression below

1. $2(a-6)=-a-13$

Solution

$2a-12=-a-13$

$2a+a=-13+12$

$3a=-1$

$a=\frac{-1}{3}$

2. $\frac{2x-3}{4}=\frac{x+1}{5}$

Solution

$5(2x-3)=4(x+1)$

$10x-15=4x+4$

$10x-4x=4+15$

$6x=19$

$x=\frac{19}{6}$

3. $(2x+1)(3x^2-x+4)$

Solution

$=2x(3x^2-x+4)+1(3x^2-x+4)$

$=(6x^3-2x^2+8x)+(3x^2-x+4)$

$=6x^3+(-2x^2+3x^2)+(8x-x)+4$

$=6x^3+x^2\ 7x+4$

4. $xy+3yx-4y^2x$

Solution

$=(xy+3yx)-4y^2x$

$=4xy-4y^2x$

TRY IT YOURSELF

EXERCISE 1.0	
i. Solve the algebraic equation below $3a + 4ba - 2a^2 + 4a^2 + 6a$	ii. Solve the algebraic equation below $a^{-1} + 2a^{-2} + 3a^{-2}$

UNDERSTANDING FACTORIZATION OF ALGEBRAIC EXPRESSIONS

Factorising is the reverse process of expanding brackets.
To factorise an algebraic expression means to put it into brackets by taking out the common factors.

Examples

Factorising

$3x + 6 = 3(x + 2)$

Expanding brackets

Factorising

$x^2 + 6x + 5 = (x + 5)(x + 1)$

Expanding brackets

PART 2 : HOW TO FACTORISE EXPRESSIONS

a. **Factorising single bracket**

Example of factorising an algebraic expression

Factorising

$3x + 6 = 3(x + 2)$

Expanding brackets

b. **Factorising double brackets**

When factorising quadratic expressions expressions in form $ax^2 + bx + c$

Factorising

$x^2 + 6x + 5 = (x + 5)(x + 1)$

Expanding brackets

Factorising

$2x^2 + 5x + 3 = (2x + 3)(x + 1)$

Expanding brackets

c. **Differences of two squares**

Using the difference of two squares

PART 3 : FACTORISATION METHODS

Each method of factorising or factoring expressions is summarised below.

1. **Factorising single brackets (example : $3x + 6$)**
   a. Find the highest common factor (HCF) of the numbers *3* (the coefficient of x) and *6* (the constant).
      The highest common factor (HCF) of $3x$ and *6* is *3*
   b. Write the highest common factor (HCF) at the front of the single bracket. $3(\quad + \quad)$
   c. Fill in each term in the bracket by multiplying out. $3(x + 2)$

2. a) **Factorising quadratics into double brackets :** $x^2 + 6x + 5$
   i. Write out the factor pairs of the last numbers *(5): 1 , 5*
   ii. Find a pair of factor that + to give the middle number (6) and $\times$ to give the last number (5) : $1 \times 5 = 5$; $1 + 5 = 6$
   iii. Write two bracket and put the variable at the start of each one $(x \quad)(x \quad)$
   iv. Write one factor in the first bracket and the other factor in the second bracket. The order isn't important, the signs of the factor

2 b) Factorising quadratics into double brackets : $2x^2 + 5x + 3$

i. Multiply the end numbers together (2 and 3) then write out the factor pairs of this new number in order *(6); 1 , 5 ,2,3*
ii. We need a pair of factor that + to give the middle middle number (5) and $\times$ to give this new number (6) : $2 + 3 = 5$; $2 \times 3 = 6$
iii. Rewrite the original expression , this time splitting the middle term into the two factor we found in step (ii) $2x^2 + 2x + 3x + 3$
iv. Split the equation down the middle and factorise fully each half. $2x(x + 1) + 3(x + 1)$
v. Factorise the whole expression by bringing whatever is in the bracket to the front and writing the two other terms in the other bracket. $(2x + 3)(x + 1)$

3. Difference of two squares : $4x^2 - 9$

a. Write down 2 bracket (   )(   )
b. Square root the first term and write it on the left-hand side of both bracket $\sqrt{4x^2} = 2x$; $(2x \quad)(2x \quad)$
c. Square root the last term and write it on the right hand side of both bracket. $\sqrt{9} = \pm 3$; $(2x \quad 3)(2x \quad 3)$
d. Put + in the middle of one bracket and – in the middle of the other $(2x + 3)(2x - 3)$

EXAMPLES

1. $p^2+6p-16$ *Solution* $=(p-2)(p+8)$	**2.** $x^2-8x=-15$ *Solution* $x^2-8x+15=0$ $(x-3)(x-5)=0$ $(x-3)=0$; $(x-5)=0$ $x=3$; $x=5$
3. $5x^2+7x-9=4x^2+x-18$ *Solution* $5x^2+7x-9-4x^2-x+18=0$ $(5x^2-4x^2)+(7x-x)+(-9+18)=0$ $x^2+6x+9=0$ $(x+3)(x+3)=0$ $x+3=0$ $x=-3$	**4.** $ax-ay+2x-2y$ *Solution* $=a(x-y)+2(x-y)$ $=(x-y)(a+2)$

DO IT YOURSELF

i. Solve the algebraic equation.	ii. Solve the algebraic equation.
$x^2-5x-10=-4$	$3-x-2x^2=0$

UNDERSTANDING SOLVING ALGEBRAIC FRACTIONS

Algebraic fractions are fractions that contain at least one variable.

Examples

$\frac{x}{12}$ — x is the numerator	$\frac{3}{x+1}$ — The denominator is an expression in terms of x	$\frac{2x}{15}$ — The numerator is a multiple of x
$\frac{x+1}{2x}$ — Both the numerator and the denominator contain an x term	$\frac{3x+4}{2x-5}$ — Both the numerator and the denominator contain an expression with x	$\frac{(3x+4)^2}{x^2-9}$ — The numerator and the denominator are quadratic expressions

PART 1 : HOW TO SOLVE EQUATIONS INCLUDING ALGEBRAIC FRACTION

In order to seolve equation including algebraic fraction:

1. **Convert each fraction so they all have a common denominator**
2. **Multiply the equation throughout by the common denominator**
3. **Solve the equation(linear or quadratic)**

UNDERSTANDING SOLVING ALGEBRAIC FRACTIONS

PART 2 : ALGEBRAIC FRACTION EXAMPLES

a. Equation with one fraction ; $\frac{2x-1}{3}+x=3$

i. Convert each fraction so they all have a common denominator. *(here we only have one fraction so not need to convert)*

ii. Multiply the equation throughout by the common denominator . *(multiply the equation throughout by 3 (the denominator)*:

$$\frac{2x-1}{3_{\times 3}}+x=3_{\times 3}$$

$$=2x-1+3x=9$$

Make sure that you multiply every term in the equation by 3

iii. Solve the equation (linear or quadratic)

$2x-1+3x=9$

$5x-1=9$

$5x=10$

$x=2$

UNDERSTANDING SOLVING ALGEBRAIC FRACTIONS

b. Equation with two fraction ; $\frac{x+1}{2}+\frac{x+3}{5}=6$

i. Convert each fraction so they all have a common denominator. *(here we only have two fraction with denominators of 2 and 5, the lowest common multiple of 2 and 5 is 10 and so can convert to the same denominator)*

$$\frac{x+1}{2} \xrightarrow{\times 5} \frac{5(x+1)}{10} = \frac{5x+5}{10}$$

$$\frac{x+3}{5} \xrightarrow{\times 2} \frac{2(x+3)}{10} = \frac{2x+6}{10}$$

We now have the equation : $\frac{5x+5}{10}+\frac{2x+6}{10}=6$

ii. Multiply the equation through out by the common denominator .*(multiply the equation throughout by 10 (the denominator)*:

$$\frac{5x+5}{10}+\frac{2x+6}{10}=6$$
$$5x+5+2x+6=60$$

Make sure that you multiply every term in the equation by 10

iii. Solve the equation (linear or quadratic)

$5x+5+2x+6=60$

$7x+11=60$

$7x=49$

$x=7$