

$$n \equiv \sum \overline{a_i} - \sum \overline{a_i} \pmod{11}$$

Example 2.14. (Kvant M 676) Prove that for every positive integer n the sum of the digits of 1981^n is not less than 19.

Proof Für $1981^n = \overline{a_k a_{k-1} \dots a_1}$

in $1981 \equiv 1 \pmod{9} \rightarrow 1981^n \equiv 1 \pmod{9}$
 $\rightarrow \sum_{i=1}^k a_i \equiv 1 \pmod{9}$

mit $\sum_{i=1}^k a_i = 1, 10, 19, \dots$

also 1981^n anheben wir 1, also $\sum_{i=1}^k a_i > 1$

sonst $\sum_{i=1}^k a_i = 10$ für $S_1 = \sum_{i=2}^k a_i$
 $S_2 = \sum_{i=1}^k a_i$

also $S_1 + S_2 = 10 \quad (1) \rightarrow S_2 = 10 - S_1$

in $1981 \equiv 9+1-8-1 = 1 \pmod{11}$

also $1981^n \equiv 1 \equiv S_2 - S_1 \pmod{11}$

also $S_2 - S_1 = 1, 12, 23, \dots$

also $S_1, S_2 > 0$ also $10 > S_2 - S_1$

also $S_2 - S_1 = 1$ in $S_2 + S_1 = 10$
 $\downarrow (1) \quad \downarrow (2)$

$(1) + (2): 2S_2 = 11$ also $S_1, S_2 \in \mathbb{N}$

$\therefore$ also in $1981^n \geq 19$ #