EXAMPLE **One Solution**

1 **Solve the system of equations.**

$5x + 3y + 2z = 2$
$2x + y - z = 5$
$x + 4y + 2z = 16$

Use elimination to make a system of two equations in two variables. First, eliminate z in the first and second equations.

$5x + 3y + 2z = 2$
$2x + y - z = 5$ $\xrightarrow{(\times 2)}$

$5x + 3y + 2z = 2$
$(+)\ 4x + 2y - 2z = 10$
$\square = \square$

Eliminate z in the first and third equations.

$5x + 3y + 2z = 2$
$(-)x + 4y + 2z = 16$
$\square = \square$

Solve the system of two equations. Eliminate y.

$9x + 5y = 12$
$4x - y = -14$ $\xrightarrow{(\times 5)}$

$9x + 5y = 12$
$(+)\ \square = \square$
$\square = \square$
$x = \square$

Substitute $\square$ for x in one of the two equations with two variables and solve for y.

$4x - y = -14$	Equation with two variables
$4(\square) - y = -14$	Replace x.
$\square - y = -14$	Multiply
$y = \square$	Simplify.

Substitute $\square$ for x and y in one of the original equations with three variables.

$2x + y - z = 5$	Equation with three variables
$2(\square) + 6 - z = 5$	Replace x and y.
$\square + 6 - z = 5$	Multiply.
$z = \square$	Simplify.

The solution is $\square$. You can check this solution in the other two original equations.

EXAMPLE **Infinite Solutions**

2 **Solve the system of equations.**

$2x + y - 3z = 5$
$x + 2y - 4z = 7$
$6x + 3y - 9z = 15$

Eliminate y in the first and third equations.

$2x + y - 3z = 5$
$6x + 3y - 9z = 15$ $\xrightarrow{(\times 3)}$

$6x + 3y - 9z = 15$
$(-)6x + 3y - 9z = 15$
$\square = \square$

The equation $0 = 0$ is always $\square$. This indicates that the first and third equations represent the same plane. Check to see if this plane intersects the second plane

$x + 2y - 4z = 7$
$6x + 3y - 9z = 15$ $\xrightarrow{(\times 6)}$

$6x + 12y - 24z = 42$
$(-)6x + 3y - 9z = 15$
$\square = \square$
$\square = \square$

The planes intersect in a $\square$. So, there are an $\square$ of solutions.

EXAMPLE **No Solution**

3 **Solve the system of equations.**

$3x - y - 2z = 4$
$6x + 4y + 8z = 11$
$9x + 6y + 12z = -3$

Eliminate x in the second two equations.

$6x + 4y + 8z = 11$ $\xrightarrow{(\times 3)}$ $\square + 12y + \square = \square$

$9x + 6y + 12z = -3$ $\xrightarrow{(\times 2)}$

$(-)18x + \square + 24z = \square$
$\square = \square$

The equation $\square = \square$ is never true. So, there is no solution of this system.