Calculate the following:

$10 \times 3^2 + 5 =$ ☐

$5^2 - 12 \div 3 \times 2^2 =$ ☐

$3 \times (4 + 5^2) =$ ☐

$2^4 \times (10 - 7) =$ ☐

$4^2 + (4 - 3)^2 =$ ☐

$(2^3 + 5 \times 2) + 6 =$ ☐

$(14 - 10)^2 \times 6 =$ ☐

$(3^2 + 3)^2 =$ ☐

$30 - (10 - 6)^2 =$ ☐

$34 + 6 \times (4^2 \div 2) =$ ☐

https://youtu.be/HZ5kHTLeuMU

Example

Consider the formula $v = u + at$. Suppose we wish to transpose this formula to obtain one for t.

Because we want to obtain t on its own we start by subtracting u from each side:

$$\begin{aligned} v &= u + at \\ v - u &= at \end{aligned}$$

We now divide everything on both sides by a.

$$\frac{v-u}{a} = \frac{at}{a} = t$$

and so finally $t = \dfrac{v-u}{a}$. We have transposed the formula to find an expression for t.

Example

Consider the formula $v^2 = u^2 + 2as$ and suppose we wish to transpose it to find u.

We want to obtain u on its own and so we begin by subtracting $2as$ from each side.

$$\begin{aligned} v^2 &= u^2 + 2as \\ v^2 - 2as &= u^2 \end{aligned}$$

Finally, taking the square root of both sides:

$$u = \sqrt{v^2 - 2as}$$

Notice we need to take the square root of the whole term $(\sqrt{v^2 - 2as})$ in order to find u.

Example

Consider the formula $s = ut + \frac{1}{2}at^2$. Suppose we want to transpose it to find a.

Because we want a on its own, we begin by subtracting ut from both sides.

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ s - ut &= \frac{1}{2}at^2 \end{aligned}$$

Multiplying both sides by 2:

$$2(s - ut) = at^2$$

Dividing both sides by t^2:

$$\frac{2(s-ut)}{t^2} = a$$

and so

$$a = \frac{2(s-ut)}{t^2}$$

Example

Suppose we wish to rearrange $y(2x+1) = x+1$ in order to find x.

Notice that x occurs both on the left and on the right. We need to try to get all the terms involving x together. We begin by expanding the brackets on the left:

$$\begin{aligned} y(2x+1) &= x+1 \\ 2xy + y &= x+1 \end{aligned}$$

Subtracting x from both sides:

$$2xy - x + y = 1$$

The left-hand side now has two terms involving x. We can factorise these as follows:

$$x(2y-1) + y = 1$$

Then subtracting y from both sides:

$$x(2y-1) = 1 - y$$

and finally, dividing both sides by $(2y-1)$

$$x = \frac{1-y}{2y-1}$$

Example

Suppose we wish to rearrange $\frac{y}{y+x} + 5 = x$ to find an expression for y.
We begin by multiplying *every term* on both sides by $(y+x)$ in order to remove the fractions:

$$y + 5(y+x) = x(y+x)$$

Next we multiply out the brackets:

$$y + 5y + 5x = xy + x^2$$

We try to get all the terms involving y onto the left-hand side. Subtracting xy from both sides:

$$6y - xy + 5x = x^2$$

Subtracting $5x$ from both sides, and taking out the common factor y we have

$$y(6-x) = x^2 - 5x$$

Finally, dividing both sides by $6 - x$ we obtain

$$y = \frac{x^2 - 5x}{(6-x)}$$

Exercise 1

Rearrange each of the following formulae to make the quantity shown the subject.

Use ^ to show power like x^2, which means x^2, use sqrt n (x) to show $\sqrt[n]{x}$ for example sqrt3(x+1) means $\sqrt[3]{(x+1)}$, if you want to show $\sqrt{x}$ write it down like sqrt(x), and use / to show division like x/2, if you want to divide x by y+2, do not write x/y+2, you must write it down like x/(y+2) .

1. $v = u + at, \quad u$	
2. $v^2 = u^2 + 2as, \quad s$	
3. $s = vt - \frac{1}{2}at^2, \quad a$	
4. $p = 2(w + h), \quad h$	
5. $A = 2\pi r^2 + 2\pi rh, \quad h$	
6. $E = \frac{1}{2}mv^2 + mgh, \quad v$	
7. $E = \frac{1}{2}mv^2 + mgh, \quad m$	
8. $a(3b - 1) = 2b + 2, \quad b$	
9. $\frac{t}{2t - s} = 3s, \quad t$	
10. $\frac{s}{2t - s} + 5 = 3t, \quad s$	

We began with the formula $T = 2\pi\sqrt{\frac{l}{g}}$. Let us now try to rearrange this to find an expression for g.

We begin by squaring both sides of the equation in order to remove the square root.

$$T^2 = (2\pi)^2\frac{l}{g}$$

To remove the fraction we multiply both sides by g:

$$T^2 g = (2\pi)^2 l$$

Dividing both sides by T^2 gives

$$g = \frac{(2\pi)^2 l}{T^2}$$

By observing the two square terms on the right, we note that this formula could be written, if we wish, in the equivalent form

$$g = \left(\frac{2\pi}{T}\right)^2 l$$

Example - the lens formula

The so-called lens formula, which is used in optics, is given by

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

Suppose we want to rearrange this formula to find u.

Because we want to isolate u we begin by subtracting $\frac{1}{v}$ from both sides.

$$\frac{1}{f} - \frac{1}{v} = \frac{1}{u}$$

The left-hand side fractions can be combined by expressing them over a common denominator

$$\frac{v-f}{fv} = \frac{1}{u}$$

Inverting both sides

$$\frac{fv}{v-f} = u$$

and so $u = \frac{fv}{v-f}$ as required.

Example

The formula $T = \dfrac{T_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$ arises in the study of relativity. Suppose we want to rearrange it to find an expression for $\dfrac{v}{c}$.

We begin by noticing that if we square both sides this will remove the square root term (i.e. the power $\frac{1}{2}$) on the right-hand side. So squaring:

$$T^2 = \frac{T_0^2}{(1 - \frac{v^2}{c^2})}$$

We remove the fraction by multiplying both sides by $\left(1 - \dfrac{v^2}{c^2}\right)$:

$$T^2\left(1 - \frac{v^2}{c^2}\right) = T_0^2$$

Dividing both sides by T^2:

$$\left(1 - \frac{v^2}{c^2}\right) = \frac{T_0^2}{T^2}$$

Adding $\dfrac{v^2}{c^2}$ to both sides gives

$$1 = \frac{T_0^2}{T^2} + \frac{v^2}{c^2}$$

Subtracting $\dfrac{T_0^2}{T^2}$ from both sides:

$$1 - \frac{T_0^2}{T^2} = \frac{v^2}{c^2}$$

Finally, taking the square root of both sides

$$\frac{v}{c} = \sqrt{1 - \frac{T_0^2}{T^2}}$$

as required.

1. $y = a + \dfrac{1}{x}, \quad x$	
2. $y = a + \dfrac{1}{1-x}, \quad x$	
3. $P = \dfrac{P_0}{1-r^2}, \quad r$	
4. $m = k\sqrt{a(1-x)}, \quad x$	
5. $V = \dfrac{V_0}{\sqrt{r^2-1}}, \quad r$	