Activity 2.2.1 Circuit Simplification: Karnaugh Mapping

INTRODUCTION

At this point you have the ability to apply the theorems and laws of Boolean algebra to simplify logic expressions to produce simpler and more cost effective digital logic circuits. You may have also realized that simplifying a logic expression using Boolean algebra, though not terribly complicated, is not always the most straightforward process. There isn't always a clear starting point to apply the various theorems and laws, nor is there a definitive end in the process.

Wouldn't it be nice to have a process for simplifying logic expressions that was more straightforward, had a clearly defined beginning, middle, and end, and didn't require you to memorize all of the Boolean theorems and laws? Well, there is, and it's called **Karnaugh mapping**. Karnaugh mapping, or K-Mapping, is a graphical technique to simplify logic expressions containing up to four variables.

In this activity you will learn how to use the Karnaugh mapping technique to simplify two-, three-, and four-variable logic expressions. Additionally, logic expressions containing **don't care conditions** will be simplified using the K-Mapping process.

Karnaugh Map
A graphical tool for finding the maximum SOP or POS simplification of a Boolean expression. A Karnaugh map works by arranging the terms of an expression so that variable scans are cancelled by grouping minterms or maxterms.

Don't Care Condition
Situation when a circuit's output level for a given set of input conditions can be assigned as either a 1 or 0.

RESOURCES

Karnaugh Mapping
Resources available online

Procedure

In your engineering notebook, write the simplified sum-of-products (SOP) logic expression for the K-Maps below.

1. $F_1 =$

	$\bar{B}$	B
$\bar{A}$	1	1
A	0	0

2 $F_2 =$

	$\bar{C}$	C
$\bar{A}\bar{B}$	0	1
$\bar{A}B$	0	0
AB	1	1
$A\bar{B}$	1	0

3 $F_3 =$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	0	1
$\bar{A}B$	0	1	0	1
AB	1	1	0	0
$A\bar{B}$	1	1	0	0

After transferring the truth table data into the K-Maps, write in your notebook the simplified sum-of-products (SOP) logic expression for the K-Maps below.

4 $F_4 =$

Q	R	F_4
0	0	0
0	1	0
1	0	1
1	1	1

	$\bar{R}$	R
$\bar{Q}$		
Q		

5 $F_5 =$

Q	R	S	F_5
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

	$\bar{S}$	S
$\bar{Q}\bar{R}$		
$\bar{Q}R$		
QR		
$Q\bar{R}$		

6 $F_6 =$

Q	R	S	T	F_6
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	1

	$\bar{S}\bar{T}$	$\bar{S}T$	ST	$S\bar{T}$
$\bar{Q}\bar{R}$				
$\bar{Q}R$				
QR				
$Q\bar{R}$				

After labeling the K-Map and transferring the truth table data into it, write in your notebook the simplified sum-of-products (SOP) logic expression for the K-Maps below.

7 $F_7 =$

W	X	F_7
0	0	0
0	1	1
1	0	1
1	1	1

8 $F_8 =$

W	X	Y	F_8
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

9 $F_9 =$

W	X	Y	Z	F_9
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	1
1	1	1	0	0
1	1	1	1	0

In your notebook, write the simplified sum-of-products (SOP) logic expression for the K-Maps below. Be sure to take advantage of any *don't care* conditions.

10 $F_{10} =$

	$\overline{L}$	L
$\overline{K}$	X	1
K	1	0

11 $F_{11} =$

	$\overline{M}$	M
$\overline{K}\,\overline{L}$	0	0
$\overline{K}\,L$	1	X
$K\,L$	1	1
$K\,\overline{L}$	X	0

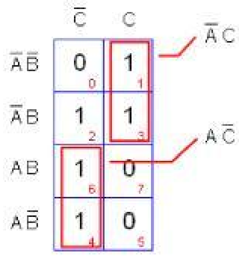
⑫ $F_{12} =$

	$\overline{M}\overline{N}$	$\overline{M}N$	MN	$M\overline{N}$
$\overline{K}\overline{L}$	0	1	1	0
$\overline{K}L$	1	1	0	X
KL	0	X	0	X
$K\overline{L}$	0	1	0	1

CONCLUSION

1. Give three advantages of using K-mapping over Boolean algebra to simplify logic expressions.
2. The three variable K-maps shown below can be completed with three groups of two. The two groups shown (cells 1 and 3; cells 4 and 6) are required. The third group needed to cover the one in cell 2, could be cells 2 and 3 or cells 2 and 6.

$F_1 = ?$



Write the two possible logic expressions for the function F1.

3. These logic expressions are considered equal and equivalent, but they do not look the same. Explain why these two expressions can be considered equal and equivalent even though they are not identical.

Going Further (Optional)

4. The following four variable K-Maps can be solved using the traditional method of grouping the 1s (identify the three groups of 8).

$F_1 = ?$

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	1	1	1	1
$\overline{A}B$	1	0	0	1
AB	1	1	1	1
$A\overline{B}$	1	1	1	1

5. Rather than taking this approach, let's get creative and take advantage of the fact that the K-Map contains only two 0s. Group these 0s and write the logic expression.

   Since you grouped the 0s, this is the logic expression for $\overline{F}_1$.

6. Now apply DeMorgan's Theorem to get the logic expression for F_1.
7. What is the advantage of taking this approach (from Question 6) over the traditional approach of circling the ones?
8. Are there any disadvantages?