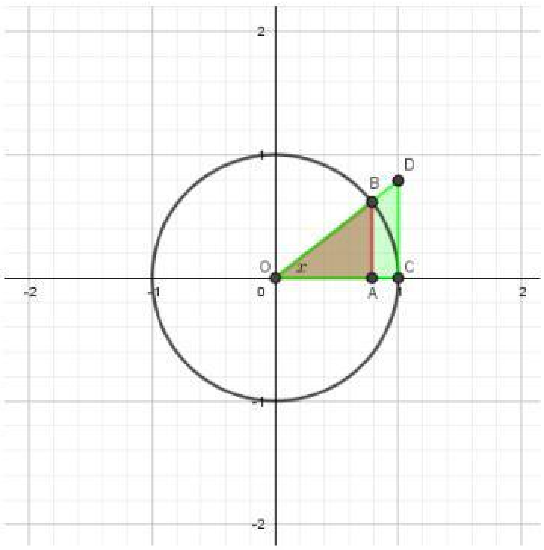
RUMUS DASAR LIMIT FUNGSI TRIGONOMETRI

Perhatikan gambar berikut!



Pada gambar di samping, diketahui,

Panjang OB = OC = 1 (jari-jari lingkaran) dan besar $\angle AOB = x$,

Maka:

1. $\sin x = \frac{AB}{OB} \leftrightarrow AB = OB \times \sin x$

   $AB = \sin x$

2. $\cos x = \frac{OA}{OB} \leftrightarrow OA = OB \times \cos x$

   $OA = \cos x$

3. $\tan x = \frac{CD}{OC} \leftrightarrow CD = OC \times \tan x$

   $CD = \tan x$

Luas ΔAOB, Luas juring COB, dan Luas ΔCOD dapat dituliskan sebagai berikut:

1. Luas $\Delta AOB = \frac{1}{2} \times OA \times AB$

   Luas $\Delta AOB = \frac{1}{2} \times \cos x \times \sin x$

2. Luas juring $COB = \frac{x}{2\pi} \times \pi \times OB^2$

   Luas juring $COB = \frac{1}{2} x$

3. Luas $\Delta COD = \frac{1}{2} \times OC \times CD$

   Luas $\Delta COD = \frac{1}{2} \tan x$

Hubungan Luas ΔAOB, Luas Juring COB, dan Luas ΔCOD

$$L.\ \Delta AOB < L.\ juring\ COB < L.\ \Delta COD$$

$$\frac{1}{2}.\cos x . \sin x \quad < \quad \frac{1}{2}x \quad < \quad \frac{1}{2}\tan x$$

$$\cos x . \sin x \quad < \quad x \quad < \quad \tan x$$

$$\cos x . \sin x \quad < \quad x \quad < \frac{\sin x}{\cos x}$$

$$\frac{\cos x . \sin x}{\sin x} < \quad \frac{x}{\sin x} \quad < \frac{\sin x}{\cos x . \sin x}$$

$$\cos x \quad < \quad \frac{x}{\sin x} \quad < \frac{1}{\cos x}$$

$$\lim_{x \to 0} \cos x \quad < \quad \lim_{x \to 0} \frac{x}{\sin x} \quad < \lim_{x \to 0} \frac{1}{\cos x}$$

$$\cos 0 < \quad \lim_{x \to 0} \frac{x}{\sin x} \quad < \frac{1}{\cos 0}$$

$$1 < \quad \lim_{x \to 0} \frac{x}{\sin x} \quad < \frac{1}{1}$$

$$1 < \quad \lim_{x \to 0} \frac{x}{\sin x} \quad < 1$$

dikarenakan

$$1 < \quad \lim_{x \to 0} \frac{x}{\sin x} \quad < 1$$

maka

Nilai $\lim_{x \to 0} \frac{x}{\sin x} = 1$

Dengan mengembangkan pola perolehan nilai

$$\lim_{x \to 0} \frac{x}{\sin x} = 1$$

Maka diperoleh rumus dasar limit fungsi trigonometri:

1) $\lim\limits_{x \to 0} \frac{\sin ax}{bx} = \frac{a}{b}$

2) $\lim\limits_{x \to 0} \frac{ax}{\sin bx} = \frac{a}{b}$

3) $\lim\limits_{x \to 0} \frac{\tan ax}{bx} = \frac{a}{b}$

4) $\lim\limits_{x \to 0} \frac{ax}{\tan bx} = \frac{a}{b}$

5) $\lim\limits_{x \to 0} \frac{\tan ax}{\tan bx} = \frac{a}{b}$

6) $\lim\limits_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$

7) $\lim\limits_{x \to 0} \frac{\tan ax}{\sin bx} = \frac{a}{b}$

8) $\lim\limits_{x \to 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}$

Tentukan nilai limit berikut!

1. $\lim\limits_{x \to 0} \frac{3x}{\tan 6x} = \frac{3.(0)}{\tan 6\,(0)} = \frac{0}{0}$ (bentuk tak tentu)

   Penggunaan rumus dasar:

   $\lim\limits_{x \to 0} \frac{3x}{\tan 6x} = \frac{}{6} = \frac{1}{}$

2. $\lim\limits_{x \to 0} \frac{sin^3 2x}{x^3} = \frac{sin^3 2(0)}{0^3} = \frac{0}{0}$ (bentuk tak tentu)

   Penggunaan rumus dasar:

   $\lim\limits_{x \to 0} \frac{sin^3 2x}{x^3} = \lim\limits_{x \to 0} \left(\frac{\sin 2x}{x}\right) = \left(\frac{}{1}\right) =$

3. $\lim\limits_{x\to 0}\frac{x\tan 3x}{sin^2 2x} \quad = \quad \frac{0\,.\tan 3\,(\quad)}{sin^2 2(0)} \quad = \quad -$

Penggunaan rumus dasar:

$$\lim_{x\to 0}\frac{x.\tan 3x}{sin^2 2x} = \lim_{x\to 0}\frac{x}{\sin 2x} \times \lim_{x\to 0}\frac{\tan 3x}{\quad}$$

$$= \quad - \quad \times \quad -$$

$$= \quad -$$

4. $\lim\limits_{x\to\frac{1}{2}}\frac{\sin(4x-2)}{\tan(2x-1)} \quad = \quad \lim\limits_{x\to\frac{1}{2}}\frac{\sin(4(\quad)-2)}{\tan(2(\quad)-1)} \quad = \quad -$

Penggunaan rumus dasar:

$$\lim_{x\to\frac{1}{2}}\frac{\sin(4x-2)}{\tan(2x-1)} = \lim_{x\to\frac{1}{2}}\frac{\sin 2(\quad x-\quad)}{\tan(2x-1)} = \quad - \quad =$$

5. $\lim\limits_{x\to 1}\frac{\sin(x-1)}{x^2-1} \quad = \quad \lim\limits_{x\to 1}\frac{\sin(\quad -1)}{\quad^2-1} \quad = \quad -$

Penggunaan rumus dasar:

$$\lim_{x\to 1}\frac{\sin(x-1)}{x^2-1} = \lim_{x\to 1}\frac{\sin(x-1)}{(\quad+\quad)(\quad-\quad)} =$$

$$= \lim_{x\to 1}\frac{\sin(x-1)}{x-1}\,.\,\lim_{x\to 1}\frac{1}{\quad}$$

$$= 1.\frac{\quad}{\quad+\quad}$$

$$= \quad -$$