**Example 2**

$f(x,y,z)=\sqrt{1-x^2-y^2-z^2}$

1) Any restriction?

2) How many restriction?

Determine the domain and the range of $f$.

**Solution**

- One restriction: $1-x^2-y^2-z^2 \geq 0$

  The domain is

  step: $-x^2-y^2-z^2 \geq -1$

Please show the steps during quizzes, tests or final exam

Domain: $D_f=\left\{(x,y,z) \,\middle|\, x,y,z \in \mathbb{R}, x^2+y^2+z^2 \leq 1\right\}$

- The range is

Range: $R_f=\left\{w \,\middle|\, w \in \mathbb{R}, 0 \leq w \leq 1\right\}$

How? Please show the steps during quizzes, tests or final exam

**FILL IN THE BLANK**

**Explanation for Example 2**

Range: $R_f=\left\{w \,\middle|\, w \in \mathbb{R}, 0 \leq w \leq 1\right\}$

How? Please show the steps during quizzes, tests or final exam

$w=\sqrt{1-x^2-y^2-z^2}$

Rewrite add bracket: $w=\sqrt{1-\left(x^2+y^2+z^2\right)}$

$x^2+y^2+z^2 \geq \square$ — Fill in the blank

min value for $x^2+y^2+z^2$ is $\square \Rightarrow w=\sqrt{1-(\quad)}=\sqrt{1}=1$

($\square$ because $x^2+y^2+z^2$ always positive)

max value for $x^2+y^2+z^2$ is $\square \Rightarrow w=\sqrt{1-(\quad)}=\sqrt{0}=0$

($\square$ because $x^2+y^2+z^2 \leq \square$ for real number)

Thus $w$ is between 0 and $1 \Rightarrow 0 \leq w \leq 1$



Prepared By Dr. Nurul Aini Jaafar, Department of Mathematical Sciences, Faculty of Science, UTM