Introduction to Vectors

Grade 10 Revision:

Scalar = a quantity that has only magnitude (size) & unit

Vector = a quantity that has magnitude, unit and direction

Vector => how much AND which way

=> represented by a line segment where length represents magnitude (i.e. drawn to scale) and arrow head represents direction

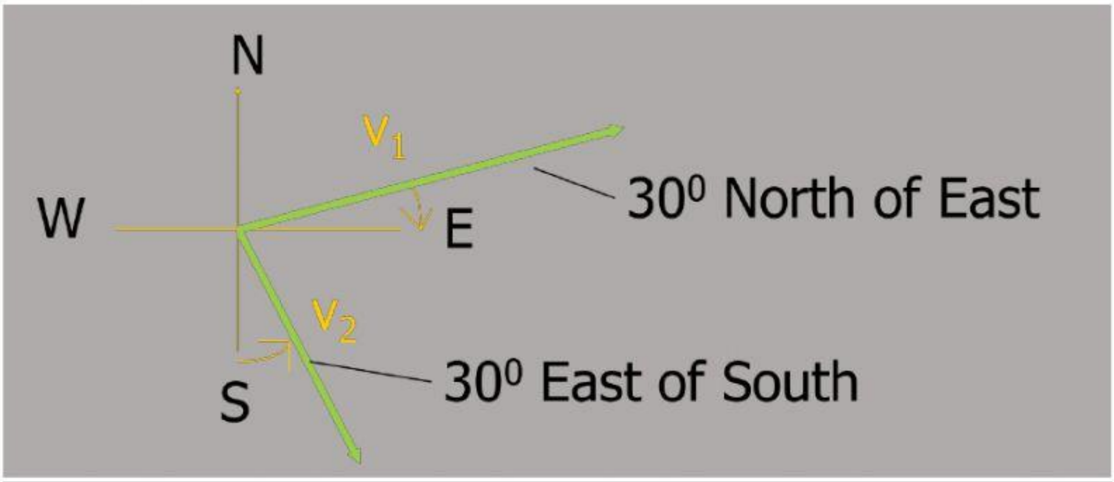
Some examples of Vectors and Scalars:

VECTORS			SCALARS		
SYMBOL	NAME	UNITS	SYMBOL	NAME	UNITS
Δx	Displacement	m	d	Distance	m
v	Velocity	ms^{-1}	v	Speed	ms^{-1}
a	Acceleration	ms^{-2}			
			t	Time	s
F	Force	N			
W	Weight	N	m	Mass	kg
			E	Energy	J
			W	Work	J
			P	Power	W
p	Momentum	$kgms^{-1}$			

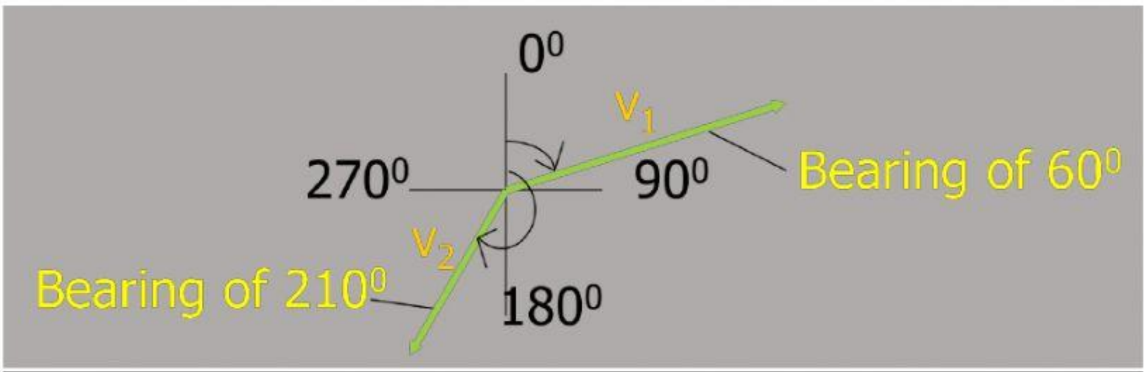
We will be moving on to study Newton's Laws and for this section, the vector we are most interested in is Force (unit: Newtons, N), so the examples we will look at will all centre around force vectors.

Vectors need a magnitude (size), a unit and a DIRECTION. In simple examples, this direction may be left, right, up or down. However, there are three main methods we use to specify direction:

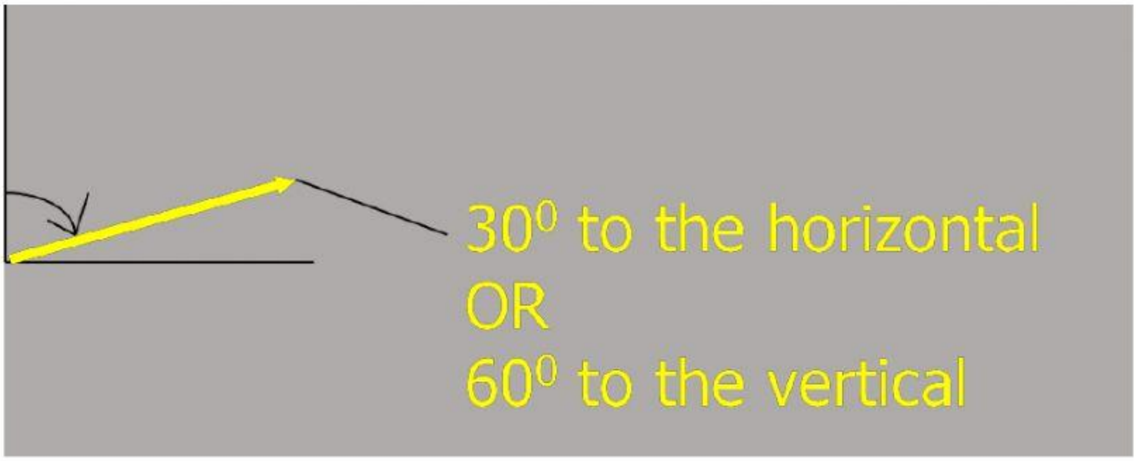
1. Compass points



2. Bearings



3. **Reference to the horizontal or vertical**



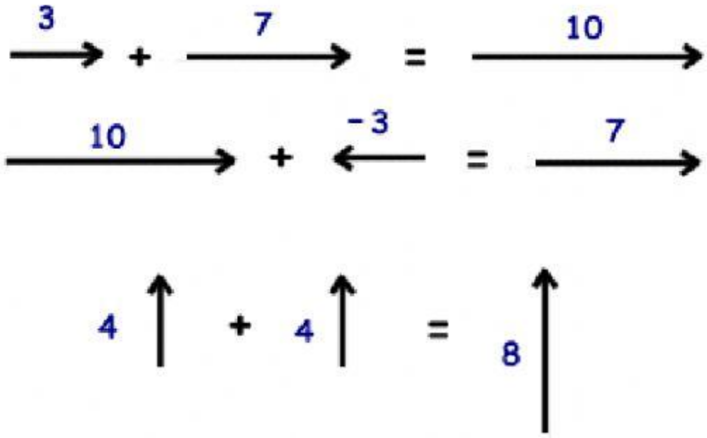
Resultant Vectors:

Resultant Vector:
The resultant of a number of vectors is the single vector having the same effect as the individual vectors together.

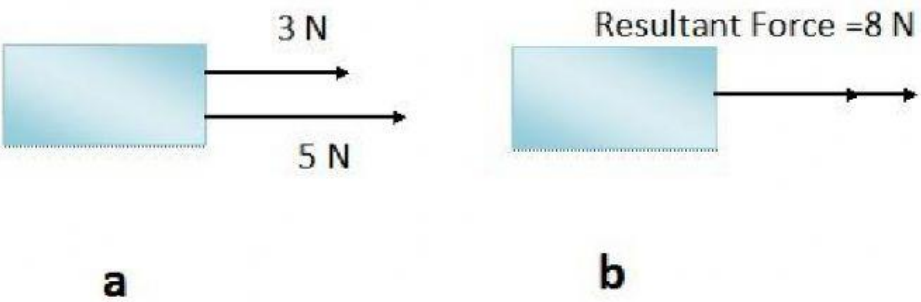
Vectors in a straight line:

- When displacements are in the same straight line, one direction is regarded as positive and the opposite direction as negative
- Vectors in the same direction are added
- Vectors in opposite directions are subtracted

e.g. (1)

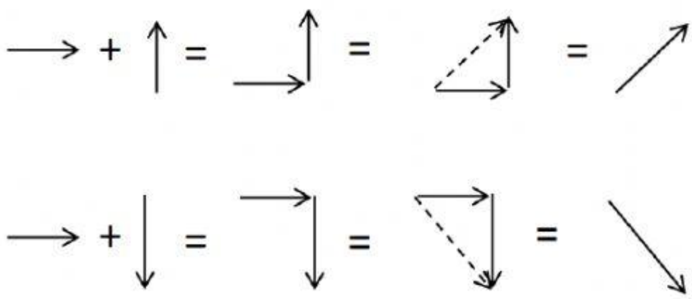


e.g. (2)



When we are working with Forces, we call this resultant force the net force, F_{net}.

<u>Vectors at an angle to each other</u>:



F_{net} (the dotted arrow) can be calculated mathematically using Pythagorus. The direction of the net force then needs to be calculated as the angle in the triangle using trig.

Remember:

$$\sin \Theta = \frac{opp}{hyp}$$

$$\cos \Theta = \frac{adj}{hyp}$$

$$\tan \Theta = \frac{opp}{adj}$$

Exercise:

Calculate the net (resultant) force of each of the following:

Hint:

- It's always helpful to draw a diagram of the forces and then decide which direction will be positive.
- We ADD all the forces together, but some forces may have a – sign due to opposite direction.
- Don't forget to include unit and direction in your answer.

1) Sarah exerts a force of 400 N on a box to the right while Tyler exerts a force of 200 N on the box to the left.

F_{net} = + =

Magnitude Unit Direction

- For direction, be guided by the question. If they use left and right, use that for your answer. If they use East and West, use that. When it gets more complicated, you will need to start using compass points, bearings and angles.
- If your answer is negative, that is telling you the direction so don't include the minus sign in the answer.

2) If Cameron exerts a force of 350 N on a box to the right but Benedict exerts a force of 100 N to the left.

F_{net} = + =

3) Both Keagan and Michaela exert a force of 400 N to the right on a trolley.

F_{net} = + =

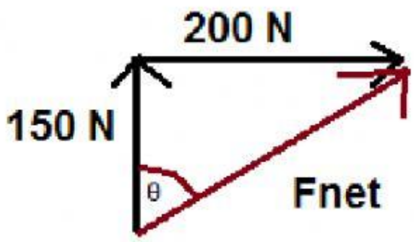
4) If Benedict exerts a force of 800 N upwards on a box and Zanele exerts a force of 350 N downwards on the box.

F_{net} = + =

5) A box is pulled up with a force of 150 N and to the right with a force of 200 N. Calculate the resultant force on the box.

First draw a force diagram:

(Note that you draw the vectors one after the other "head to tail")



Then use Pythagoras to calculate Fnet:

$Fnet^2 = x^2 + y^2$

$= \quad ^2 + \quad ^2$

=

Fnet = N (square root to calculate this answer)

Then calculate the angle of Fnet using trig ratios:

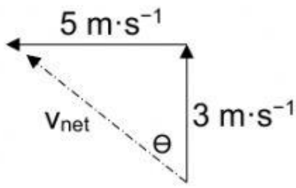
Sin θ = $\frac{o}{h}$	Cos θ = $\frac{a}{h}$	Tan θ = $\frac{o}{a}$

Tan θ = $\frac{o}{a}$

$=\frac{200}{150}$ = (cos or sin could also be used to calculate this angle)

ϴ = ° (Note: to find ϴ from tan ϴ use the inverse tan function on calculator ($\tan^{-1}$))

6) A frog is trying to cross a river. It swims at 3 $m \cdot s^{-1}$ in a northerly direction towards the opposite bank. The water is flowing in a westerly direction at 5 $m \cdot s^{-1}$. Find the frog's resultant velocity by using appropriate calculations.



$v_{net}^2 = \quad ^2 + \quad ^2$

$=$

$v_{net} = \quad m.s^{-1}$

The direction could be expressed as angle "ϴ west of north".

To calculate ϴ: $\tan \Theta = \frac{opp}{hyp} = \frac{5}{3} =$

ϴ = °

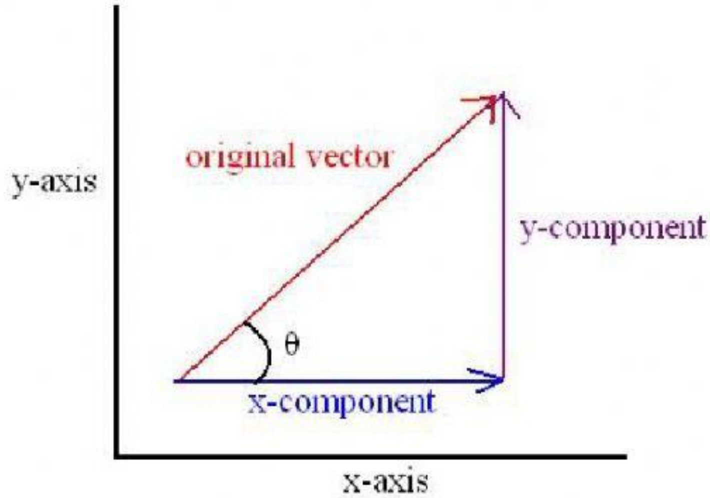
The final answer would be expressed as:

$m.s^{-1}$ at an angle of ° west of north

Components of Vectors:

Components:
2 or more vectors which can replace a single vector

Just as 2 vectors can be combined into a net or resultant vector, so a single vector can be resolved into components. A vector at an angle can be split into an x-component (horizontal) and a y-component (vertical).



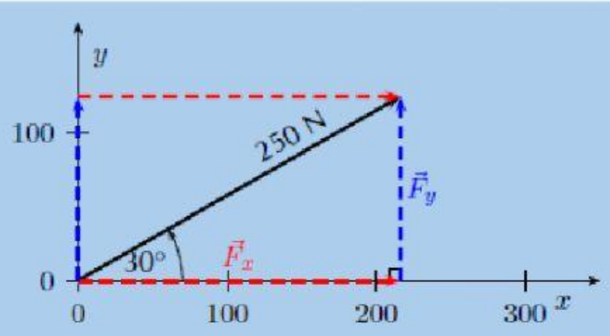
Because the x and y components are at 90˚ to each other, trig can be used to calculate the components of a single vector. If the original vector is F, then we would call the x-component F_x and the y-component F_y. The components can be calculated as follows:

$$\cos \Theta = \frac{F_x}{F} \qquad \boxed{F_x = F \cos \Theta}$$

$$\sin \Theta = \frac{F_y}{F} \qquad \boxed{F_y = F \sin \Theta}$$

Example

A force of 250 N acts at an angle of 30° to the positive x-axis. Resolve this force into components parallel to the x- and y-axes.



Notice how the two components acting together give the original vector as their resultant.

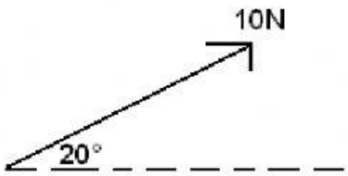
Now we can use trigonometry to calculate the magnitudes of the components of the original displacement:

$$F_y = 250\sin(30^\circ) = 125 \text{ N}$$

and

$$F_x = 250\cos(30^\circ) = 216{,}51\text{N}$$

Calculate the horizontal and vertical components of the following:



F_x = F cos Θ

= cos

= N

F_y = F sin Θ

= sin

= N