Thruth Tables

A truth table shows how the truth or falsity of a compound statement depends on the truth or falsity of the simple statements from which it's constructed. So we'll start by looking at truth tables for the five logical connectives. Here's the table for negation:

p	¬p
V	F
F	V

This table is easy to understand. If P is true, its negation $\neg P$ is false. If P is false, then $\not P$ is true.

$P \wedge Q$ should be true when both P and Q are true, and false otherwise:

p	q	p ∧ q
V	V	V
V	F	F
F	V	F
F	F	F

$P \vee Q$ is true if either P is true or Q is true (or both — remember that we're using "or" in the inclusive sense). It's only false if both P and Q are false.

p	q	p ∨ q
V	V	V
V	F	V
F	V	V
F	F	F

Here's the table for logical implication:

p	q	p → q
V	V	V
V	F	F
F	V	V
F	F	V

To understand why this table is the way it is, consider the following example:

"If you get an A, then I'll give you a dollar."

The statement will be true if I keep my promise and false if I don't. Suppose it's true that you get an A and it's true that I give you a dollar. Since I kept my promise, the implication is true. This corresponds to the first line in the table.

Suppose it's true that you get an A but it's false that I give you a dollar. Since I didn't keep my promise, the implication is false. This corresponds to the second line in the table.

What if it's false that you get an A? Whether or not I give you a dollar, I haven't broken my promise. Thus, the implication can't be false, so (since this is a two-valued logic) it must be true. This explains the last two lines of the table.

$P \iff Q$ means that P and Q are equivalent. So the double implication is true if P and Q are both true or if P and Q are both false; otherwise, the double implication is false.

p	q	p ↔ q
V	V	V
V	F	F
F	V	F
F	F	V

You should remember (or be able to construct) the truth tables for the logical connectives. You'll use these tables to construct tables for more complicated sentences. It's easier to demonstrate what to do than to describe it in words, so you'll see the procedure worked out in the examples.

Remark:

When you're constructing a truth table, you have to consider all possible assignments of True (T) and False (F) to the component statements. For example, suppose the component statements are P, Q, and R. Each of these statements can be either true or false, so there are $2^3 = 8$ possibilities.

p	q	r
V	V	V
V	V	F
V	F	V
V	F	F
F	V	V
F	V	F
F	F	V
F	F	F

Example. Construct a truth table for the formula $\neg P \wedge (P \rightarrow Q)$.

1. First, I list all the alternatives for P and Q.
2. Next, in the third column, I list the values of $\neg P$ based on the values of P. I use the truth table for negation: When P is true $\neg P$ is false, and when P is false, $\neg P$ is true.
3. In the fourth column, I list the values for $P \rightarrow Q$. Check for yourself that it is only false ("F") if P is true ("T") and Q is false ("F").
4. The fifth column gives the values for my compound expression $\neg P \wedge (P \rightarrow Q)$. It is an "and" of $\neg P$ (the third column) and $P \rightarrow Q$ (the fourth column). An "and" is true only if both parts of the "and" are true; otherwise, it is false. So I look at the third and fourth columns; if both are true ("T"), I put T in the fifth column, otherwise I put F.

p	q	¬p	p → q	¬p∧(p → q)
V	V	F	V	F
V	F	F	F	F
F	V	V	V	V
F	F	V	V	V

Tautology and Contradiction

A **tautology** is a formula which is "always true" — that is, it is true for every assignment of truth values to its simple components. You can think of a tautology as a rule of logic.

The opposite of a tautology is a **contradiction**, a formula which is "always false". In other words, a contradiction is false for every assignment of truth values to its simple components.
Example. Show that $(P \rightarrow Q) \vee (Q \rightarrow P)$ is a tautology.

Exercises

p	q	¬p	¬p → q	p∧(¬p → q)
V	V			
V	F			
F	V			
F	F			

p	q	¬p	¬p → q	p → (¬p → q)
V				
V				
F				
F				

Let p, q, r and s represent the following simple statements:

$p : 7^2 = 49$.

$q : \sqrt{49} = 6$.

r : A rectangle does not have 4 sides .

s : A triangle has 3 sides.

Write each sentence below in symbolic form. Then determine its truth value.

1. $7^2 = 49$ and $\sqrt{49} = 6$
2. $7^2 = 49$ or $\sqrt{49} \neq 6$
3. A rectangle does not have 4 sides or a triangle has 3 sides.
4. If a rectangle does not have 4 sides, then a triangle has 3 sides.
5. If $7^2 = 49$, then $\sqrt{49} = 6$.
6. If $7^2 \neq 49$, then a rectangle does not have 4 sides.