

MATEMATIKA PEMINATAN

KELAS XII

Nama :

Kelas/No. :

Kompetensi Dasar : 3.3 Menggunakan prinsip turunan ke fungsi trigonometri

4.3 Menyelesaikan masalah yang berkaitan dengan turunan fungsi trigonometri

A. Definisi Turunan

Turunan fungsi $f(x)$ didefinisikan sebagai:

$$f'(x) = \frac{df(x)}{dx} = \frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

B. Dasar Turunan Fungsi Aljabar

1. $f(x) = x^n$, maka $f'(x) = \dots x \dots$
2. $f(x) = ax^n$, maka $f'(x) = \dots \dots x \dots$

C. Sifat-sifat Turunan Fungsi Aljabar

Jika diketahui k suatu konstanta, $u = u(x)$, $v = v(x)$, dan masing-masing mempunyai turunan $u'(x)$ dan $v'(x)$, maka berlaku:

1. $f(x) = u + v$, maka $f'(x) = u' + v'$
2. $f(x) = u - v$, maka $f'(x) = \dots - \dots$
3. $f(x) = uv$, maka $f'(x) = u' \cdot \dots + \dots \cdot \dots$
4. $f(x) = f(u)$, maka $f'(x) = f'(u) \cdot u'$
5. $f(x) = \frac{u}{v}$, maka $f'(x) = \frac{\dots \cdot \dots - \dots \cdot \dots}{\dots \dots}$

D. Mengingat Kembali Turunan pada Fungsi Aljabar

1. $f(x) = 5$
 $f'(x) = \dots$
2. $f(x) = x$
 $f'(x) = \dots$
3. $f(x) = -3x^2$
 $f'(x) = \dots$
4. $f(x) = (3x)^2$
 $f'(x) = \dots (\dots) \dots$
 $f'(x) = \dots$

$$5. f(x) = 5(3x^2 - 7x)^4$$

$$f'(x) = \dots \dots (\dots - \dots) \dots (\dots - \dots)$$

$$f'(x) = \dots (\dots - \dots) (\dots - \dots) \dots$$

E. Turunan Fungsi Trigonometri

1. Turunan dari $y = f(x) = \sin x$

Diketahui $f(x) = \sin x$ maka $f(x+h) = \sin(\dots + \dots)$

$$\begin{aligned} f'(x) &= \frac{dy}{dx} = \frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\dots (\dots + \dots) - \dots}{\dots} \\ &= \lim_{h \rightarrow 0} \frac{\dots \dots \left(\frac{\dots + \dots + \dots}{2} \right) \sin \left(\frac{\dots + \dots - \dots}{2} \right)}{\dots} \\ &= \dots \lim_{h \rightarrow 0} \frac{\dots \left(\frac{\dots + \dots}{2} \right) \sin \left(\frac{\dots}{2} \right)}{\dots} \\ &= \dots \lim_{h \rightarrow 0} \frac{\dots \left(\frac{\dots + \dots}{2} \right) \sin \left(\frac{1}{2}h \right)}{\dots} \\ &= \dots \lim_{h \rightarrow 0} \dots \left(\frac{\dots + \dots}{2} \right) \lim_{h \rightarrow 0} \frac{\dots (- \dots)}{\dots} \\ &= \dots \dots \left(\frac{\dots + \dots}{2} \right) \frac{\dots / \dots}{\dots} \\ &= \dots \cdot \cos \frac{\dots}{2} \cdot \frac{\dots}{\dots} \\ &= \dots \cdot \frac{\dots}{\dots} \cdot \cos \dots \\ &= \dots \end{aligned}$$

Jadi turunan dari $f(x) = \sin x$ adalah $f'(x) = \cos x$

2. Turunan dari $y = f(x) = \cos x$

Diketahui $f(x) = \cos x$ maka $f(x+h) = \cos(\dots + \dots)$

$$\begin{aligned}
 f'(x) &= \frac{dy}{dx} = \frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\dots (\dots + \dots) - \dots}{\dots} \\
 &= \lim_{h \rightarrow 0} \frac{\dots \dots \left(\frac{\dots + \dots + \dots}{2} \right) \sin \left(\frac{\dots + \dots - \dots}{2} \right)}{\dots} \\
 &= \dots \lim_{h \rightarrow 0} \frac{\dots \left(\frac{\dots + \dots}{2} \right) \sin \left(\frac{\dots}{2} \right)}{\dots} \\
 &= \dots \lim_{h \rightarrow 0} \frac{\dots \left(\frac{\dots + \dots}{2} \right) \sin \left(\frac{1}{2}h \right)}{\dots} \\
 &= \dots \lim_{h \rightarrow 0} \dots \left(\frac{\dots + \dots}{2} \right) \lim_{h \rightarrow 0} \frac{\dots (-\dots)}{\dots} \\
 &= \dots \dots \left(\frac{\dots + \dots}{2} \right) \frac{\dots / \dots}{\dots} \\
 &= \dots \cdot \sin \frac{\dots}{2} \cdot \frac{\dots}{\dots} \\
 &= \dots \cdot \frac{\dots}{\dots} \cdot \sin \dots \\
 &= \dots
 \end{aligned}$$

Jadi turunan dari $f(x) = \cos x$ adalah $f'(x) = -\sin x$

3. $f(x) = x + \cos x$

$$f'(x) = \dots + (\dots)$$

$$f'(x) = \dots - \dots$$

4. $f(x) = 3 \sin x - 4 \cos x$

$$f'(x) = \dots - \dots (\dots)$$

$$f'(x) = \dots + \dots$$

5. $f(x) = x \cdot \sin x$

$$u = x, u' = \dots$$

$$v = \dots, v' = \dots$$

$$f'(x) = \dots \cdot \dots + \dots \cdot \dots$$

$$f'(x) = \dots + \dots \dots$$

6. $f(x) = x^2 \cdot \cos x$

$$u = x^2, u' = \dots$$

$$v = \dots, v' = \dots$$

$$f'(x) = \dots \cdot \dots + \dots^2 \cdot (\dots)$$

$$f'(x) = \dots \dots - \dots^2 \dots$$

7. $f(x) = \sin x \cdot \cos x$

$$u = \dots, u' = \dots$$

$$v = \dots, v' = \dots$$

$$f'(x) = \dots \cdot \dots + \dots \cdot (\dots)$$

$$f'(x) = \dots \dots \dots - \dots \dots \dots$$

$$f'(x) = \dots \quad (\text{identitas trigonometri})$$

8. $f(x) = \frac{\sin x}{\cos x}$

$$u = \dots, u' = \dots$$

$$v = \dots, v' = \dots$$

$$f'(x) = \frac{\dots \cdot \dots - \dots \cdot (\dots)}{(\dots)^2}$$

$$f'(x) = \frac{\dots \dots \dots + \dots \dots \dots}{\dots \dots \dots}$$

$$f'(x) = \frac{\dots}{\dots \dots \dots}$$

$$f'(x) = \dots \dots \dots$$