

Lembar Kerja Peserta Didik (LKPD)

Topik: Operasi Vektor di R^3

Nama :

Kelas :

No Absen :

Operasi Vektor di R^3

Jika dua vektor $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ dan $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ adalah vektor-vektor tidak nol di R^3 , maka

- Penjumlahan vektor

$$\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}$$

- Pengurangan vektor

$$\vec{a} - \vec{b} = (a_1 - b_1)\hat{i} + (a_2 - b_2)\hat{j} + (a_3 - b_3)\hat{k}$$

- Perkalian titik (dot product)

a. Jika tidak diketahui sudut antara dua vektor: $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

b. Jika diketahui sudut antara dua vektor: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \alpha$

- Perkalian silang (cross product)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - a_3b_2)\hat{i} - (a_1b_3 - a_3b_1)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$$

- Sudut antara dua vektor: $\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

Latihan: Selesaikan soal berikut

- Diketahui $\vec{a} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$ dan $\vec{b} = \begin{pmatrix} 1 \\ 10 \\ 2 \end{pmatrix}$. Hitunglah:
 - $3\vec{a} + 2\vec{b}$
 - $2\vec{a} - 3\vec{b}$
- Diketahui $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ dan $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$. Hitunglah:
 - Perkalian titik (dot product) : $\vec{a} \cdot \vec{b}$
 - Perkalian silang (cross product) : $\vec{a} \times \vec{b}$
- Diketahui $\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ dan $\vec{b} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. Hitunglah besar sudut antara $\vec{a}$ dan $\vec{b}$

Jawab:

No	Penyelesaian
1a	Diketahui $\vec{a} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$ dan $\vec{b} = \begin{pmatrix} 1 \\ 10 \\ 2 \end{pmatrix}$ $3\vec{a} + 2\vec{b} = 3\begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} + 2\begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} + \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$
1b	Diketahui $\vec{a} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$ dan $\vec{b} = \begin{pmatrix} 1 \\ 10 \\ 2 \end{pmatrix}$ $2\vec{a} - 3\vec{b} = 2\begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} - 3\begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} - \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix} = \begin{pmatrix} \dots \\ \dots \\ \dots \end{pmatrix}$
2a	$\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k} \rightarrow a_1 = \dots ; a_2 = \dots ; a_3 = \dots$ $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k} \rightarrow b_1 = \dots ; b_2 = \dots ; b_3 = \dots$ $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = (\dots)(\dots) + (\dots)(\dots) + (\dots)(\dots)$ $= \dots + \dots + \dots$ $= \dots$ Jadi, perkalian titik (dot product) $= \vec{a} \cdot \vec{b} = \dots$
2b	$\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k} \rightarrow a_1 = \dots ; a_2 = \dots ; a_3 = \dots$ $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k} \rightarrow b_1 = \dots ; b_2 = \dots ; b_3 = \dots$ $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2) \hat{i} - (a_1 b_3 - a_3 b_1) \hat{j} + (a_1 b_2 - a_2 b_1) \hat{k}$ $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{vmatrix} = ((\dots)(\dots) - (\dots)(\dots)) \hat{i} - ((\dots)(\dots) - (\dots)(\dots)) \hat{j} + ((\dots)(\dots) - (\dots)(\dots)) \hat{k}$

$$= ((.....) - (.....)) \hat{i} - ((.....) - (.....)) \hat{j} + ((.....) - (.....)) \hat{k}$$

$$= (.....) \hat{i} - (.....) \hat{j} + (.....) \hat{k}$$

Perkalian silang (cross product) : $\vec{a} \times \vec{b} = (.....) \hat{i} - (.....) \hat{j} + (.....) \hat{k}$

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Diketahui $\vec{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \rightarrow a_1 =, a_2 =, a_3 =$

dan $\vec{b} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \rightarrow b_1 =, b_2 =, b_3 =$

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{(\dots\dots)^2 + (\dots\dots)^2 + (\dots\dots)^2} = \sqrt{\dots\dots + \dots\dots + \dots\dots} = \sqrt{\dots\dots}$$

$$|\vec{b}| = \sqrt{b_1^2 + b_2^2 + b_3^2} = \sqrt{(\dots\dots)^2 + (\dots\dots)^2 + (\dots\dots)^2} = \sqrt{\dots\dots + \dots\dots + \dots\dots} = \sqrt{\dots\dots}$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = (.....)(.....) + (.....)(.....) + (.....)(.....)$$

$$= + +$$

$$=$$

$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{.....}{(\sqrt{.....})(\sqrt{.....})} = \frac{.....}{\sqrt{.....}} = \frac{.....}{.....} = \text{ (tiga angka di belakang koma)}$$

$$\alpha = \arccos =^\circ \text{ (satu angka di belakang koma)}$$

Jadi, besar sudut antara vektor $\vec{a}$ dan $\vec{b}$ adalah $^\circ$ (satu angka di belakang koma)