

PEMBAHASAN LATIHAN SOAL EHB-BKS
BAB PERSAMAAN TRIGONOMETRI
SOAL NOMOR 14 - 17

Rumus persamaan Trigonometri

- $\sin x = \sin \alpha$
 - $\rightarrow x = \alpha + k \cdot 360^\circ$
 - $\rightarrow x = (180^\circ - \alpha) + k \cdot 360^\circ$
- $\cos x = \cos \alpha$
 - $\rightarrow x = \alpha + k \cdot 360^\circ$
 - $\rightarrow x = -\alpha + k \cdot 360^\circ$
- $\tan x = \tan \alpha$
 - $\rightarrow x = \alpha + k \cdot 180^\circ$

k = bil. bulat sembarang

MATERI PRASYARAT

	90°	
	Sin (+)	Semua (+)
180°	Kuadran II	Kuadran I
	Kuadran III	Kuadran IV
	Tan (+)	Cos (+)
	270°	

$$\begin{aligned}\sin A + \sin B &= 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \\ \sin A - \sin B &= 2 \cos \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right) \\ \cos A + \cos B &= 2 \cos \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right) \\ \cos A - \cos B &= -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)\end{aligned}$$

1. $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
2. $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
3. $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
4. $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
5. $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \cdot \tan \beta}$
6. $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \cdot \tan \beta}$
7. $\sin 2\alpha = 2 \sin \alpha \cos \alpha$
8. $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$
 $\cos 2\alpha = 2 \cos^2 \alpha - 1$
 $\cos 2\alpha = 1 - 2 \sin^2 \alpha$
9. $\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$
10. $\sin^2 \alpha = \frac{1}{2} - \frac{1}{2} \cos 2\alpha$
11. $\cos^2 \alpha = \frac{1}{2} + \frac{1}{2} \cos 2\alpha$
12. $\sin 3\alpha = 3 \sin \alpha - 4 \sin^3 \alpha$

14. Sesuaikan pernyataan-pernyataan di bawah ini dengan memberi tanda centang pada kolom yang sudah disediakan.

PERNYATAAN	SESUAI
Diketahui $\sin \alpha = \frac{1}{5}\sqrt{13}$, α sudut lancip. Nilai $\cos 2\alpha = -\frac{1}{25}$	☐
Selesaian dari $\cos 2x - \sin x = 0$ dengan $0^\circ \leq x \leq 360^\circ$ adalah 30°	☐
Diketahui $(A + B) = \pi/2$ dan $\sin A \sin B = 1/4$. Nilai dari $\cos (A - B) = \frac{1}{2}$	☐

PEMBAHASAN:

$$\begin{aligned}\cos 2a &= 1 - 2 \sin^2 a && (\text{rumus 8 : } \cos 2a = 1 - 2\sin^2 a) \\ &= 1 - 2 \left(\frac{1}{5}\sqrt{13} \right)^2 \\ &= 1 - \frac{26}{25} \\ &= -\frac{6}{25}\end{aligned}$$

Jadi pilihan pertama ...

$$\begin{aligned}\cos 2x - \sin x &= 0 && (\text{rumus 8 : } \cos 2a = 1 - 2\sin^2 a) \\ 1 - 2\sin^2 x - \sin x &= 0\end{aligned}$$

$$0 = 2\sin^2 x + \sin x - 1 \quad (\text{pindah ruas})$$

$$0 = (2 \sin x - \dots)(\sin x + \dots) \quad (\text{faktorkan})$$

$$2 \sin x - \dots = 0$$

$$\sin x = \frac{\dots}{2}$$

$$\boxed{\text{Ingat } \frac{1}{2} = \sin \dots}$$

bernilai positif, berarti di K_I dan K_{II}

$$\text{di } K_I \rightarrow x = \dots$$

$$\text{di } K_{II} \rightarrow x = 180 - \dots = \dots$$

$$\sin x + \dots = 0$$

$$\sin x = -\dots$$

$$\boxed{\text{Ingat } 1 = \sin \dots}$$

bernilai negatif, berarti di K_{III} dan K_{IV}

$$\text{di } K_{III} \rightarrow x = (180 + \dots) = \dots$$

$$\text{di } K_{IV} \rightarrow x = (360 - \dots) = \dots$$

Maka $x = \dots^\circ$ atau $x = \dots^\circ$ atau $x = \dots^\circ$

Jadi pilihan kedua ...

$$(A + B) = \pi/2 \text{ atau}$$

$$(A + B) = 90$$

maka

$$\cos (A + B) = \cos 90$$

$$\cos A \cos B - \sin A \sin B = \cos 90 \quad (\text{rumus 3 : } \cos (A+B) = \cos A \cos B - \sin A \sin B)$$

$$\cos A \cos B - \sin A \sin B = \dots$$

$$\cos A \cos B = \sin A \sin B$$

$$\cos A \cos B = \dots$$

$$\cos (A - B) = \cos A \cos B + \sin A \sin B$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{\sqrt{3} + 1}{2}$$

Jadi pilihan ketiga

15. Berilah centang pada pernyataan-pernyataan yang benar di bawah ini dan boleh lebih dari satu jawaban.

☐ $2\cos x + 1 = 0, 0^\circ \leq x \leq 180^\circ$ nilai x yang memenuhi adalah 60°

☐ Solusi x yang memenuhi $\sqrt{3}\tan x - 1 = 0, 0^\circ \leq x \leq 180^\circ$ adalah 30°

☐ Himpunan penyelesaian dari $\cos^2 x - 2\cos x - 3 = 0, 0^\circ \leq x \leq 360^\circ$ adalah $\{180^\circ\}$

☐ Selesaian yang memenuhi $\sin 3x + \sin x = 0, 0^\circ \leq x \leq 360^\circ$ adalah 90°

PEMBAHASAN:

$$2\cos x + 1 = 0$$

$$2\cos x = -1$$

$$\cos x = -\frac{1}{2} \quad (\text{bernilai negatif) berarti ada di } K_{II} \text{ dan } K_{III}$$

$$\text{Ingat } \frac{1}{2} = \cos 60^\circ$$

$$\text{di } K_{II} \rightarrow x = (90^\circ - 60^\circ) = 30^\circ$$

$$\text{di } K_{III} \rightarrow x = (90^\circ + 60^\circ) = 150^\circ$$

Jadi pilihan pertama ...

$$\sqrt{3}\tan x = 1$$

$$\tan x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad (\text{bernilai positif) berarti ada di } K_{I} \text{ dan } K_{III}$$

$$\text{Ingat } \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\text{di } K_{I} \rightarrow x = 30^\circ$$

$$\text{di } K_{III} \rightarrow x = (90^\circ + 30^\circ) = 120^\circ$$

Jadi pilihan kedua ...

$$\cos^2 x - 2\cos x - 3 = 0 \quad (\text{faktorkan})$$

$$(\cos x - 3)(\cos x + \dots) = 0$$

$$\cos x - \dots = 0$$

$$\cos x = \dots$$

tidak mungkin karena

$\cos x$ bernilai antara 1 dan -1

$$\cos x + \dots = 0$$

$$\cos x = -\dots \quad (\text{bernilai negatif, berarti di } K_{III} \text{ \& } K_{IV})$$

Ingat $1 = \cos \dots$

$$\text{di } K_{III} \rightarrow x = (\dots - \dots) = \dots$$

$$\text{di } K_{IV} \rightarrow x = (\dots + \dots) = \dots$$

Jadi pilihan ketiga ...

$$\sin 3x + \sin x = 0, 0^\circ \leq x \leq 360^\circ$$

Gunakan rumus

$$\sin A + \sin B = 2 \sin \left(\frac{A+B}{2} \right) \cos \left(\frac{A-B}{2} \right)$$

$$\sin 3x + \sin x = 2 \sin \left(\frac{3x+x}{2} \right) \cos \left(\frac{3x-x}{2} \right)$$

$$= 2 \sin \left(\frac{4x}{2} \right) \cos \left(\frac{2x}{2} \right)$$

$$= 2 \sin 2x \cos x$$

$$\sin 3x + \sin x = 0$$

$$2 \sin 2x \cos x = 0$$

$$\sin 2x \cos x = \frac{0}{\dots}$$

$$\sin 2x \cos x = \dots$$

$$\sin 2x = 0$$

$$\sin 2x = \sin \dots$$

$$2x = \dots + k \cdot 360$$

$$x = \dots + k \cdot \dots$$

$$k=0 \rightarrow x = \dots$$

$$k=1 \rightarrow x = \dots$$

$$k=2 \rightarrow x = \dots$$

$$2x = (180 - \dots) + k \cdot 360$$

$$x = \dots + k \cdot \dots$$

$$k=0 \rightarrow x = \dots$$

$$k=1 \rightarrow x = \dots$$

$$\cos x = 0$$

bernilai positif

Jadi di K_I dan K_{IV}

Ingat $0 = \cos \dots$

$$\text{di } K_I \rightarrow x = \dots$$

$$\text{di } K_{IV} \rightarrow x = 360 - \dots = \dots$$

$$\text{Jadi } x = \dots^0 \text{ atau } x = \dots^0 \text{ atau } x = \dots^0 \text{ atau } x = \dots^0 \text{ atau } x = \dots^0$$

Jadi pilihan keempat ...

16. Nilai x yang memenuhi persamaan trigonometri $\cos 4x - \cos 2x = 0$, $0^\circ \leq x \leq 90^\circ$ adalah...

PEMBAHASAN:

$$\cos 4x - \cos 2x = 0, 0^\circ \leq x \leq 90^\circ$$

Gunakan rumus :

$$\cos A - \cos B = -2 \sin \left(\frac{A+B}{2} \right) \sin \left(\frac{A-B}{2} \right)$$

$$\begin{aligned}\cos 4x - \cos 2x &= -2 \sin \left(\frac{4x+2x}{2} \right) \sin \left(\frac{4x-2x}{2} \right) \\ &= -2 \sin \left(\frac{6x}{2} \right) \sin \left(\frac{2x}{2} \right) \\ &= -2 \sin(3x) \sin(x)\end{aligned}$$

Maka

$$\begin{aligned}\cos 4x - \cos 2x &= 0 \\ -2 \sin 3x \sin x &= 0 \\ \sin 3x \sin x &= \frac{0}{-2} \\ \sin 3x \sin x &= 0\end{aligned}$$

$\swarrow$ $\sin 3x = 0$ $\sin 3x = \sin \dots$ $\swarrow$ $3x = \dots + k \cdot 360$$x = \dots + k \cdot \dots$$k=0 \rightarrow x = \dots$ $3x = (180 - \dots) + k \cdot 360$$x = \dots + k \cdot \dots$$k=0 \rightarrow x = \dots$	$\searrow$ $\sin x = 0$ $\sin x = \sin \dots$ $x = 0$ (hanya $x = 0$ yang memenuhi di $0^\circ \leq x \leq 90^\circ$)
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Jadi $x = \dots^0$ atau $x = \dots^0$ atau $x = \dots^0$