Unit 1 Section 4 : Other Bases

The ideas that we have considered can be extended to other number bases.

The table lists the digits used in some other number bases.

Base	Digits Used
2	0, 1
3	0, 1, 2
4	0, 1, 2, 3
5	0, 1, 2, 3, 4

The powers of the base number give the place values when you convert to base 10. For example, for base 3, the place values are the powers of 3, i.e. 1, 3, 9, 27, 81, 243, etc.

This is shown in the following example, which also shows how the base 3 number 12100 is equivalent to the base 10 number 144.

Base 3

$$81 \quad 27 \quad 9 \quad 3 \quad 1$$

$$1 \quad 2 \quad 1 \quad 0 \quad 0 \quad \rightarrow \quad (1 \times 81) + (2 \times 27) + (1 \times 9) + (0 \times 3) + (0 \times 1) = 144 \text{ in base 10}$$

The following example shows a conversion from base 5 to base 10 using the powers of 5 as place values.

Base 5

$$625 \quad 125 \quad 25 \quad 5 \quad 1$$

$$4 \quad 1 \quad 0 \quad 0 \quad 1 \quad \rightarrow \quad (4 \times 625) + (1 \times 125) + (0 \times 25) + (0 \times 5) + (1 \times 1) = 2626 \text{ in base 10}$$

Example 1

Convert each of the following numbers to base 10:

(a)   412 in base 6

(b)   374 in base 9

(c)   1432 in base 5

$(4 \times 36) + (1 \times 6) + (2 \times 1) = 152$

$(3 \times 81) + (7 \times 9) + (4 \times 1) = 310$

$(1 \times 125) + (4 \times 25) + (3 \times 5) + (2 \times 1) = 242$

Example 2

Convert each of the following base 10 numbers to the base stated:

(a)   472 to base 4 $(1 \times 256) + (3 \times 64) + (1 \times 16) + (2 \times 4) + (0 \times 1) = 13120$

(b)   179 to base 7 $(3 \times 49) + (4 \times 7) + (4 \times 1) = 344$

(c)   342 to base 3 $(1 \times 243) + (1 \times 81) + (0 \times 27) + (2 \times 9) + (0 \times 3) + (0 \times 1) = 110200$

Example 3

Carry out each of the following calculations in the base stated:

(a)	$14 + 21$	base 5	40
(b)	$16 + 32$	base 7	51
(c)	$141 + 104$	base 5	300
(d)	$212 + 121$	base 3	1110

Check your answer in (a) by changing to base 10 numbers

$9 + 11 = 20$

Example 4

Carry out each of the following multiplications in the base stated:

(a)	141×23	base 5	4343
(b)	122×12	base 3	10011
(c)	512×24	base 6	21532

Check your answer to (b) by converting to base 10 numbers

$17 \times 5 = 85$

Question 1

Convert the following numbers from the base stated to base 10:

(a) 412   base 5   [ ]

(b) 333   base 4   [ ]

(c) 728   base 9   [ ]

(d) 1210   base 3   [ ]

(e) 1471   base 8   [ ]

(f) 612   base 7   [ ]

(g) 351   base 6   [ ]

(h) 111   base 3   [ ]

Question 2

Convert the following numbers from base 10 to the base stated:

(a)   24   to base 3   [_____]

(b)   16   to base 4   [_____]

(c)   321   to base 5   [_____]

(d)   113   to base 6   [_____]

(e)   314   to base 7   [_____]

(f)   84   to base 9   [_____]

(g)   142   to base 3   [_____]

(h)   617   to base 5   [_____]

Question 3

Carry out the following additions in the base stated:

(a) $3 + 2$ in base 4   [_____]

(b) $5 + 8$ in base 9   [_____]

(c) $4 + 6$ in base 8   [_____]

(d) $2 + 2$ in base 3   [_____]

(e) $6 + 7$ in base 9   [_____]

(f) $3 + 4$ in base 6   [_____]

Question 4

In what number bases could each of the following numbers be written:

(a) 123   Base: [_____]

(b) 112   Base: [_____]

(c) 184   Base: [_____]

Question 5

Carry out each of the following calculations in the base stated:

(a) $13 + 23$ in base 4   [_____]

(b) $120 + 314$ in base 5   [_____]

(c) $222 + 102$ in base 3   [_____]

(d) $310 + 132$ in base 4   [_____]

(e) $624 + 136$ in base 7   [_____]

(f) $211 + 142$ in base 5   [_____]

(g) $333 + 323$ in base 4   [_____]

(h) $141 + 424$ in base 5   [_____]

Check your answers to parts (a), (c) and (e) by converting to base 10 numbers.

(a) [_____] + [_____] = [_____]

(c) [_____] + [_____] = [_____]

(e) [_____] + [_____] = [_____]

Question 6

Carry out each of the following multiplications in the base stated:

(a) 3×2 in base 4 [____]

(b) 4×3 in base 5 [____]

(c) 4×2 in base 6 [____]

(d) 3×5 in base 6 [____]

(e) 2×2 in base 3 [____]

(f) 8×8 in base 9 [____]

Question 7

Carry out each of the following multiplications in the base stated:

(a) 121×11 in base 3 [____]

(b) 133×12 in base 4 [____]

(c) 13×24 in base 5 [____]

(d) 142×14 in base 5 [____]

(e) 161×24 in base 7 [____]

(f) 472×32 in base 8 [____]

(g) 414×22 in base 5 [____]

(h) 2101×21 in base 3 [____]

Check your answers to parts (a), (c) and (e) by converting to base 10 numbers.

(a) [　　] × [　　] = [　　]

(c) [　　] × [　　] = [　　]

(e) [　　] × [　　] = [　　]

Question 8

In which base was each of the following calculations carried out?

(a) $4 + 2 = 11$ Base: [　　]

(b) $7 + 5 = 13$ Base: [　　]

(c) $8 × 2 = 17$ Base: [　　]

(d) $4 × 5 = 32$ Base: [　　]

(e) $11 - 3 = 5$ Base: [　　]

(f) $22 - 4 = 13$ Base: [　　]

Question 9

(a) Change 147 in base 8 into a base 3 number. [　　]

(b) Change 321 in base 4 into a base 7 number. [　　]

(c) Change 172 in base 9 into a base 4 number. [　　]

(d) Change 324 in base 5 into a base 6 number. [　　]

Question 10

In which base was each of the following calculations carried out?

(a) $171 \times 12 = 2272$ Base: ⬚

(b) $122 \times 21 = 11102$ Base: ⬚

(c) $24 \times 32 = 1252$ Base: ⬚

(d) $333 \times 33 = 23144$ Base: ⬚