

Octal and Hexadecimal Number Systems

OCTAL or **BASE-8** numbers uses eight symbols: 0, 1, 2, 3, 4, 5, 6, and 7 (count them!) and position plays a major role in expressing their meaning. For example $53,702_8$ means

$$\frac{5 \times 8^4}{4096\text{'s}} + \frac{3 \times 8^3}{512\text{'s}} + \frac{7 \times 8^2}{\text{Sixty-fours}} + \frac{0 \times 8^1}{\text{Eights}} + \frac{2 \times 8^0}{\text{Ones (Units)}}$$

To change this number to base 10, multiply each placeholder by the amount its location represents and add: $(5 \times 4096) + (3 \times 512) + (7 \times 64) + (0 \times 8) + (2 \times 1) = 20,480 + 1536 + 448 + 0 + 1 = 22,466_{10}$

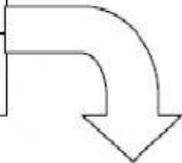
Now you try some:

$$436_8 = \text{_____} (\text{base } 10)$$

$$1234_8 = \text{_____} (\text{base } 10)$$

$$524_8 = \text{_____} (\text{base } 10)$$

| | | | | | | |
|---------|----|----|----|----|----|----|
| Base 16 | A | B | C | D | E | F |
| Base 10 | 10 | 11 | 12 | 13 | 14 | 15 |



HEXADECIMAL or **BASE-16** numbers uses sixteen symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, and E (count them!) and position plays a major role in expressing their meaning. For example $537CA_{16}$ means

$$\frac{5 \times 16^4}{65,536\text{'s}} + \frac{3 \times 16^3}{4096\text{'s}} + \frac{7 \times 16^2}{256\text{'s}} + \frac{C \times 16^1}{\text{Sixteens}} + \frac{A \times 16^0}{\text{Ones (Units)}}$$

To change this number to base 10, multiply each placeholder by the amount its location represents and add: $(5 \times 65,536) + (3 \times 4096) + (7 \times 256) + (12 \times 8) + (10 \times 1) = 327,680 + 12,288 + 1792 + 96 + 10 = 341,866_{10}$

Now you try some:

$$486_{16} = \text{_____} (\text{base } 10)$$

$$1234_{16} = \text{_____} (\text{base } 10)$$

$$EDA_{16} = \text{_____} (\text{base } 10)$$

Changing a Decimal Number to an Octal Number

Repeatedly divide by eight and record the remainder for each division – read “answer” upwards.

Example: Rewrite the decimal number 215_{10} as an octal number.

$$\begin{array}{r} 8 \overline{) 215} \\ 8 \overline{) 26} \quad R=7 \quad \circ \quad \circ \quad \circ \\ 8 \overline{) 3} \quad R=2 \\ 8 \overline{) 0} \quad R=3 \quad \uparrow \text{read} \uparrow \\ 0 \end{array}$$

8 divides into 215 twenty-six times with a remainder of 7; then 8 divides into 26 three times with a remainder of 2; and so forth...

The octal result is read upwards \uparrow , therefore
 $215_{10} = 327_8$

Now you try one:

$$682_{10} = \frac{\quad}{8}$$

Changing a Decimal Number to a Hexadecimal Number

Repeatedly divide by sixteen and record the remainder for each division – read “answer” upwards.

Example: Rewrite the decimal number 215_{10} as an octal number.

$$\begin{array}{r} 16 \overline{) 215} \\ 16 \overline{) 13} \quad R=7 \quad \circ \quad \circ \\ 16 \overline{) 0} \quad R=13_{10} = D \\ 0 \quad \uparrow \text{read} \uparrow \end{array}$$

16 divides into 215 thirteen times with a remainder of 7; then 16 divides into 13 zero times with a remainder of 13, which is represented in base

The octal result is read upwards \uparrow , therefore
 $215_{10} = D7_{16}$

Now you try one:

$$1682_{10} = \frac{\quad}{16}$$

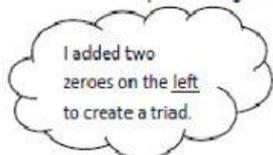
Note how the above algorithms can be adapted to change a decimal number to any chosen base.

Changing Bases Back and Forth between Binary, Octal, and Hexadecimal Systems: An Easy Task!

1. From Binary to Octal – Count off from right to left by three and translate each triad into base 10. These digits will be the base-8 symbols to express this binary number in octal.
2. From Binary to Hexadecimal - Count off from right to left by four and translate each quad into base 10. These digits will be the base-16 symbols to express this binary number in hexadecimal.
3. From Hexadecimal OR Octal to Binary – Change each symbol to binary and you are done!
4. From Octal to Hexadecimal OR from Hexadecimal to Octal – Change the higher base to binary and then use #1 or #2 above to change the binary number to the base desired.

EXAMPLES:

a) Change 1101001010_2 to an octal number.



001 101 001 010
 ↓ ↓ ↓ ↓
 1 5 1 2

therefore, the octal number is 1512_8

b) Change 1001011101_2 to a hexadecimal number.

0010 0101 1101

2 5 13/D

therefore, the hexadecimal number is $25D_{16}$

c) Change $A3D9_{16}$ to a binary number.

A 3 D 9

1010 0011 1101 1001

therefore, the binary number is 1010001111011001_2

d) Change 630076_8 to a binary number.

6 3 0 0 7 6

110 011 000 000 111 110

therefore, the binary number is

110011000000111110_2

e) Change $A45_{16}$ to octal.

A 4 5
 1010 0100 0101
 101 001 001 101
 5 1 1 5

(rewritten in binary)

(regrouped the binary digits into groups of three)

therefore the octal number is 5115_8

j) Change 5401_8 to hexadecimal.

| | | | | |
|------|------|------|-----|---|
| 5 | 4 | 0 | 1 | |
| 101 | 100 | 000 | 001 | (rewritten in binary) |
| 1011 | 0000 | 0001 | | (regrouped the binary digits into groups of four) |
| B | 0 | 1 | | therefore the hexadecimal number is B01₁₆ |

Further Exercises

- Express each number as a decimal number.
 - 263_8
 - $B21_{16}$
 - 5100_8
 - $100E_{16}$
 - 100332_8
 - 10011_{16}
- Express each number as a binary number.
 - 2524_8
 - $BAC9_{16}$
 - 332210_8
 - $4009D_{16}$
- Express each number as an octal number.
 - 101001001_2
 - 1001010000100010_2
 - $B78_{16}$
 - 1234_{16}
- Express each number as a hexadecimal number.
 - 1010100000010101010_2
 - 1010101010_2
 - 2526_8
 - 50004734_8