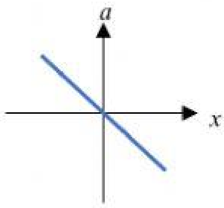
Simple Harmonic Motion

Simple harmonic motion is defined as the motion of a particle such that its acceleration, a, is proportional to displacement, x, from a fixed point and is directed towards the fixed point. In equation form, this can be written as:

$$a = -\omega^2 x$$

where ω^2 is a constant. The minus sign in the equation must be preserved and hence we take the constant as a squared quantity because this will ensure that the constant is always positive (because the square of both positive and negative number are positive). The negative sign shows that the acceleration, a, is always in the opposite direction to the displacement, x, i.e. the acceleration is always directed towards the fixed point from which the displacement is measured. If the equation is represented in a graph of a against x, it will be a straight line passing through origin, with negative gradient (due to the minus sign in the equation).



Newton's second law states that the force acting on a body is proportional to the acceleration of the body. Since the acceleration of the particle is proportional to its displacement, the resultant force acting on the particle is also proportional to the displacement and is always acting towards the fixed point (the same as acceleration). We call this as restoring force and it is experienced by object experiencing simple harmonic motion. It is also the only external force acting on a particle undergoing free oscillations.

At equilibrium position, the displacement of the pendulum is 0 and the maximum displacement of the pendulum is x_0.

We knew from Kinematics that velocity can be found by finding the gradient of a displacement-time graph. We can see from the graph above that at maximum displacement, x_0, the gradient is zero and when the displacement = 0, the gradient is maximum. Therefore:

- when the pendulum is at maximum displacement the velocity, $v = 0$, and
- when the pendulum is at equilibrium position the velocity is maximum, v_0.

And then we had also learnt from Kinematics that the gradient of a velocity-time graph can be used to find the acceleration, a. At maximum speed, v_0, the gradient is zero and when the velocity = 0, the gradient is maximum. Therefore, the pendulum has:

- largest acceleration when $v = 0$, while it is at its maximum displacement and
- $a = 0$ when the velocity is maximum, while the pendulum is at equilibrium position.