# Production Profits

Did you know that some companies only produce a profit for certain levels of customers? For example, a gym could be profitable if they had 2–10 customers because they could hire fewer employees, purchase and maintain fewer machines, and occupy a smaller space. From 11–40 customers, the gym actually loses money because they have to hire more employees, purchase and maintain more machines, and rent a larger space. Once the gym hits 41 customers though, their profits continue to grow ever higher.

This real-world context is one that can be explained with a quadratic function. In order to solve a quadratic function and determine these essential features and more, we may need to complete the square.

If Tina's lemonade stand has the profit function as defined by:

$p(x) = x^2 - 14x + 45$, where $p(x)$ represents the profit and $x$ represents the number of customers

At the moment, only the y-intercept is immediately obvious from looking at the function. \$45 would be the profit Tina has with 0 customers.

If we change the form of the function by factoring via the x-method, we would see the profit function changed to:

$p(x) = (x - 9)(x - 5)$

We factored this by seeing what factors of 45 would add to the middle term, −14. If we solve each group, we will have our solutions; $x = 9$ and $x = 5$. This means that the x-intercepts are 9 and 5. From 0 to 5 customers, Tina's lemonade stand is profitable while more than 9 customers also yields a positive profit. For example, $p(10) = 5$.

But what if Tina wanted to know how low her profit would sink in between those two levels of customers? If we complete the square, we can determine the vertex of this function which will give us the minimum extreme.

$x^2 - 14x + 45 = 0$; $x^2 - 14x + ? = -45 + ?$; $x^2 - 14x + 49 = -45 + 49$

$(x - 7)^2 = 4$; $(x - 7)^2 - 4 = 0$ ← This is now in vertex form. The vertex is $(7, -4)$ meaning that at 7 customers, Tina will lose \$4! Since the axis of symmetry goes through the vertex, we also know that value is $x = 7$.

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Directions:

The different forms of a quadratic function allow us to identify different, key features. For example, standard form gives us the y-intercept, factored form gives us the zeros, and completing the square gives us the vertex and axis of symmetry.

Translating between the different forms allows us to interpret quadratic functions within different contexts as well. We'll use these forms in order to critically think about what the features mean in given contexts.

1.) Janice's pet sitting business has the profit function

$p(x) = x^2 - 10x + 21;$, where $p(x)$ represents the profit and $x$ represents the number of pets

What is the y-intercept and what does it represent in the context of this problem? Use a complete sentence.

_____

_____

_____

_____

2.) Factor Janice's profit function:

Answer: (  )(  )

3.) What are the zeros of the function?

$x =$ _____ ; $x =$ _____

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4.) What do the zeros represent? Use a complete sentence.

5.) Complete the square for Janice's profit function.

Answer: $p(x) =$ _____

6.) From the 'complete the square' step above, identify the vertex and axis of symmetry.

Vertex: (  ,  )  Axis of Symmetry: $x =$ _____

7.) Interpret the vertex in the context of the problem. Use a complete sentence.

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