MODULE 3 FINANCE AND MODELLING

QUESTION 1

Lolwethu inherits a R1 000 000 from a long-lost, wealthy relative! To supplement her income she decides to invest the money in a living annuity at 6,4% per annum, compounded quarterly.

She will withdraw R30 000 per quarter from the living annuity, starting immediately. Determine, in years and months, how long her inheritance will last.

[12]

$$1\ 000\ 000 - 30\ 000 = \frac{30\ 000\left[1-\left(1+\frac{0,064}{4}\right)^{-n}\right]}{\frac{0,064}{4}}$$

$$0,482\ 666 = 1,016^{-n}$$

$n = 45,890$ quarters

QUESTION 2

Wayne's parents are planning for his final years of schooling in 2033, 2034 and 2035. They project the annual fees to be R28 000, R30 400 and R33 000 for the three years respectively.

The first payment into a savings account is to be made on 1 January 2017 and the last payment on 1 January 2035. They plan to deposit R270 each month into an account that earns 5,2% interest per annum, compounded monthly. Money will be withdrawn on 1 January 2033, 1 January 2034 and 1 January 2035 to pay for his fees.

Show that they assumed the annual (compound) inflation rate would be more or less constant during 2033 and 2034. (6)

$$33\ 000 = 30\ 400(1 + r)$$
$$30\ 400 = 28\ 000(1 + r)$$

Calculate the total value to which their monthly payments will accrue on 1 January 2035. (6)

$$\frac{270\left[\left(1+\frac{0,052}{12}\right)^{18\times12+1}-1\right]}{\frac{0,052}{12}}$$

Determine the total interest earned by the savings account over the full period of the investment. (4)

$$96\ 928,20 - 270 \times (18 \times 12 + 1)$$

Determine whether the savings account will be sufficient to cover the annual fees for all three years. Show your calculations. (8)

$$33\ 000 + 30\ 400\left(1+\frac{0,052}{12}\right)^{12} + 28000\left(1+\frac{0,052}{12}\right)^{24}$$

QUESTION 3

Calculate the values of the next three terms in the sequence described by the second order recursive formula:

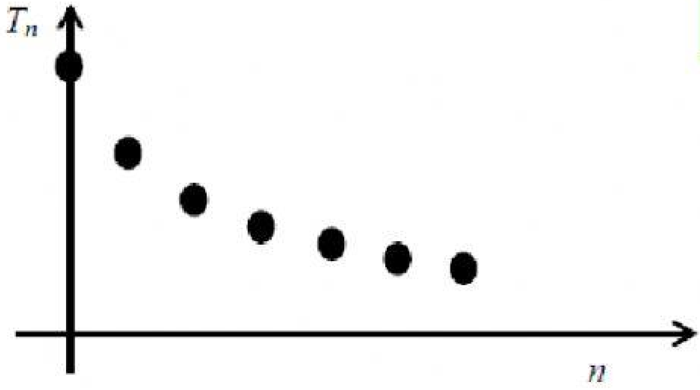
$T_{n+1} = 4.T_n + 3.T_{n-1} - 4(-1)^{n-1}$ with $T_1 = -T_2 = -2$ (5)

$T_3 = 4(2) + 3(-2) - 4(-1)^{2-1}$

$T_4 = 4(-2) + 3(2) - 4(-1)^{3-1}$

$T_5 = 4(2) + 3(-2) - 4(-1)^{4-1}$

A Malthusian model is described by the recursive formula $T_{n+1} = p.T_n$ with $0 < p < 1$. Draw a graph of T_n against n (with n the independent variable) that represents the population trend for this model as n increases.



A Logistic model has a carrying capacity of 120 and an initial population of 54. **Two** cycles later the population has reached 70. Calculate the intrinsic growth rate of the model, correct to two decimal digits. (9)

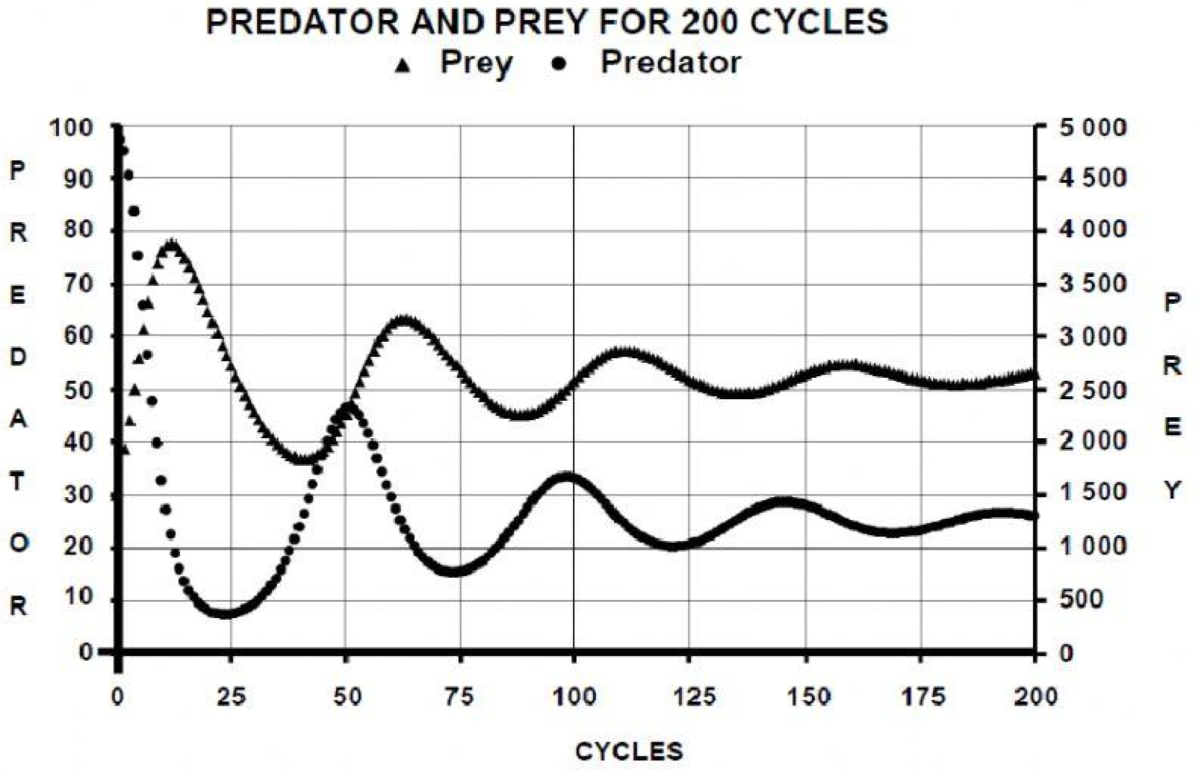
$$P_{n+1} = \overbrace{54 + r \cdot 54\left(1 - \frac{54}{120}\right)} =$$

$$P_{n+2} = P_{n+1} + r \cdot P_{n+1}\left(1 - \frac{P_{n+1}}{120}\right)$$

$$70 = (54 + 29,7r) + r(54 + 29,7r)\left(1 - \frac{54 + 29,7r}{120}\right)$$

QUESTION 4

The accompanying graph represents 200 cycles of the populations for a particular predator and prey relationship, based on the Lotka-Volterra model.



This is a discrete population model. How can this be seen in the graph, and what does 'discrete' mean mathematically? (2)

Give the approximate range of the prey population as an interval. (2)

Approximately, how many cycles after each peak of the prey population does the predator population peak?

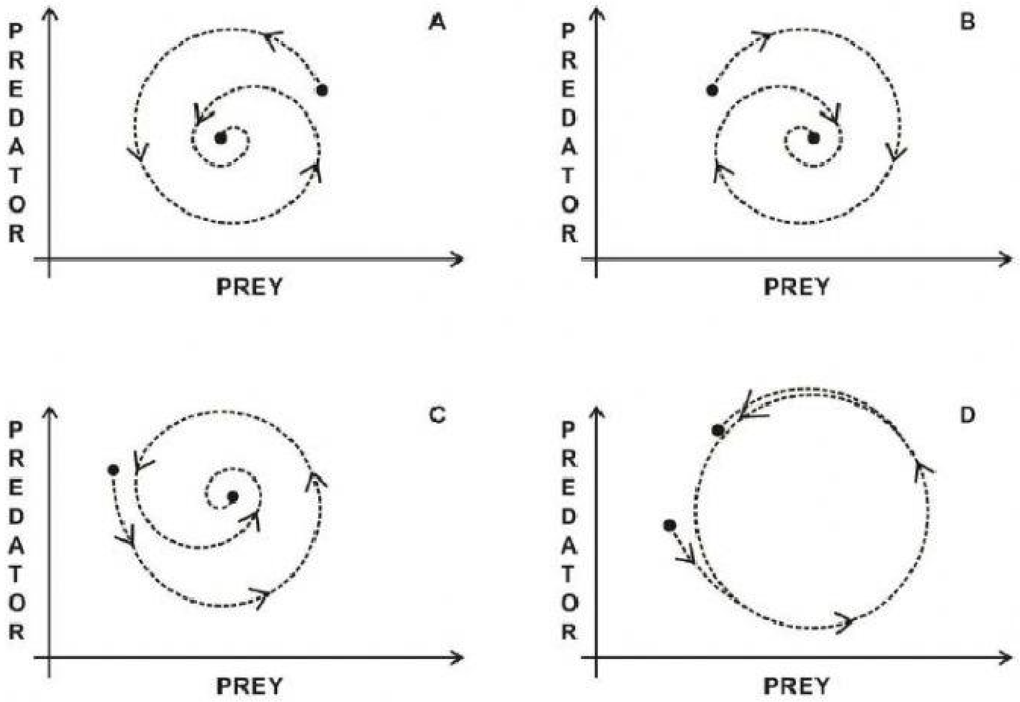
In the predator population, which is the greater: the rate of increase from a minimum to a maximum, or the rate of decrease from a maximum to a minimum? Give a reason for your answer. (2)

Refer to the Lotka-Volterra formulae in the **Information Booklet**:

(a) Which of the parameters a, b or K need to be decreased so that the prey population increases? Give a reason for your answer. (3)

(b) The parameter c is increased. What impact could this have on the prey population? Explain your answer fully in words. (3)

Which of the phase plots best represent this particular model? Explain your choice.



(4)

[18]