GRADE 12 EXAMINATION

ADVANCED PROGRAMME MATHEMATICS: PAPER II

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 Given $\begin{pmatrix} 2 & 8 & 3 \\ 5 & 4 & 1 \end{pmatrix}\begin{pmatrix} 4 & 1 \\ y & 3 \\ x & 4 \end{pmatrix} = \begin{pmatrix} 38 & 38 \\ 46 & 21 \end{pmatrix}$, solve for x and y. (7)

1.2 To solve the following equations simultaneously,

$$x + 2y - z = -1$$
$$2x + 6y + z = 7$$
$$5x + 7y - 4z = 9$$

we apply Gaussian row-reduction to the augmented matrix, $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 2 & 6 & 1 & 7 \\ 5 & 7 & -4 & 9 \end{array}\right)$

Step 1: $R_2 - 2R_1$ $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 2 & 3 & 9 \\ 5 & 7 & -4 & 9 \end{array}\right)$

Step 2: $R_3 - 5R_1$ $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 2 & 3 & 9 \\ 0 & -3 & 1 & 14 \end{array}\right)$

Step 3: $R_1 - R_2$ $\left(\begin{array}{ccc|c} 1 & 0 & -4 & -10 \\ 0 & 2 & 3 & 9 \\ 0 & -3 & 1 & 14 \end{array}\right)$

Continue the **Gaussian row reduction**, to solve for the variables x, y and z. (8)

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QUESTION 2

2.1 Given, $P = \begin{bmatrix} 2 & 2 & 6 \\ 4 & 2 & 2 \end{bmatrix}$, calculate P', the image of P, using only matrix algebra,

(a) Translated one unit left and three units down.

(b) Reflected in the $y = -x$.

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2.2 Find a simplified single matrix that first enlarges a shape by a factor of $\frac{2}{\sqrt{2}}$ and then rotates it 225°. (8)

2.3 A line with end points (3; 4) and (–1; 0) is reflected to points $\left(\frac{-3+4\sqrt{3}}{2};\frac{4+3\sqrt{3}}{2}\right)$ and $\left(\frac{1}{2};\frac{-\sqrt{3}}{2}\right)$ respectively. Find the gradient of the line of reflection. (12)
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QUESTION 3

3.1 State whether the given statements are true or false, giving a reason if false.

(a) In a given square matrix, if you multiply a row or column by a non-zero constant, the determinant is multiplied by that same non-zero constant. (2)

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(b) If you interchange two columns in a determinant, the resulting determinant will be the same. (2)

(c) The determinant of a matrix will be zero if a row is a constant multiple of another row. (2)

3.2 Given $A = \begin{pmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{pmatrix}$

Joe produced the following first step in calculating the determinant of a 4×4 matrix.

$$\begin{vmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{vmatrix} = p\begin{vmatrix} 4 & 1 & 2 \\ 3 & 2 & 1 \\ 9 & 3 & 1 \end{vmatrix} + q\begin{vmatrix} 3 & 0 & 1 \\ 4 & 1 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

(a) Joe applied his knowledge of determinants to considerably simplify the process of finding the determinant. Describe what property of determinants Joe applied to produce his first step. (2)

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(b) Determine the value of p and q.

(c) Hence, using matrix algebra, show **that the determinant is 24.**

QUESTION 4

Select the most correct answer in each question:

4.1 What is the number of edges present in a complete graph with n vertices?

(a) $\frac{n(n+1)}{2}$

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(b) $\frac{n(n-1)}{2}$

(c) n

(d) The information given is insufficient

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