GRADE 12 EXAMINATION

ADVANCED PROGRAMME MATHEMATICS: PAPER II

QUESTION 3

A financial investment can be described by the following recurrence relation, where F is given in rand and n is the number of years.

$F_n = aF_{n-1} + b; \quad F_0 = 15\,000$

It is also given that $F_1 = 26\,200$ and $F_2 = 38\,296$

3.1 Interpret the meaning of b and F_0 in the context of the investment. (2)

3.2 Calculate the values of a and b respectively. (6)

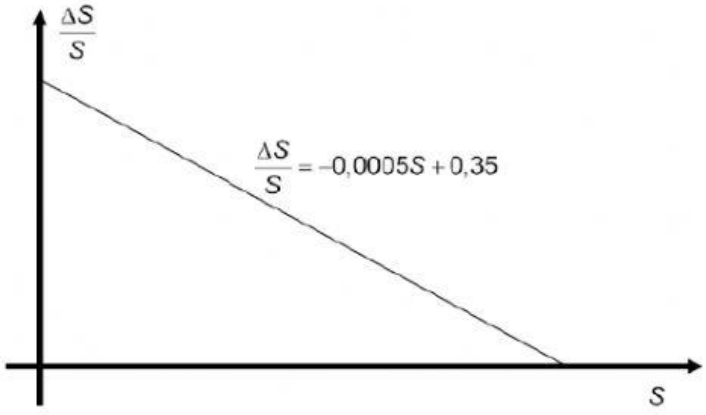
3.3 State the interest rate. (2)

[10]

LIVEWORKSHEETS

QUESTION 4

Sandile rears sheep. He allows them to reproduce naturally, subject to the constraints of the farm environment. There are no predators but space and food resources would prevent them from continuing to increase in numbers exponentially. Sandile predicts that the sheep will increase in number according to the following model, illustrated graphically:



4.1 What type of population growth is Sandile illustrating? (1)

4.2 What is the maximum number of sheep expected? (3)

4.3 What is the intrinsic growth rate? (2)

4.4 Write the growth model as a recursive formula:

$S_{n+1} = \ldots$ where n is the number of years. (4)

LIVEWORKSHEETS

4.5 If Sandile starts with 50 sheep, after how many years will the sheep population exceed half of the carrying capacity of the environment? (2)

[12]

QUESTION 5

5.1 In a certain area of the African savanna, lions prey mainly on the zebra population. Since a ban on poaching is strictly applied, the lions have no predators themselves. The lion and zebra populations can be modelled effectively using the Lotka-Volterra recursive formulae:

$$L_{n+1} = L_n + f.bZ_nL_n - cL_n$$

$$Z_{n+1} = Z_n + aZ_n\left(1 - \frac{Z_n}{K}\right) - bZ_nL_n$$

The lions and zebras have now reached stable populations with approximately 1 000 zebras being hunted by eight lions.

Records suggest that a lion encountering a zebra results in a kill occurring in 5% of the cases.

The intrinsic growth rate of the zebra population is known to be 0,8 (80%) per annum.

(a) Explain the meaning of the term bZ_nL_n. (2)

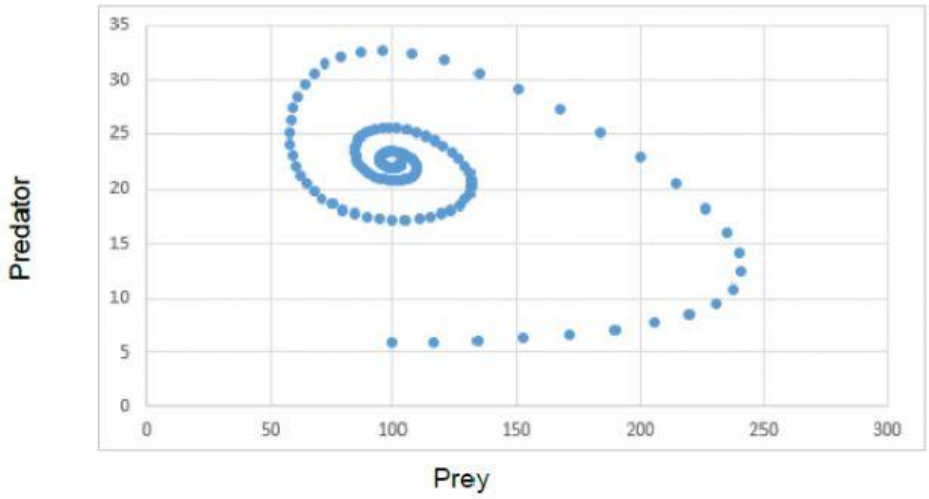
LIVEWORKSHEETS

(b) If the stable populations of lions and zebras are given by L_E and Z_E respectively, prove that:

$$L_E = \frac{a}{b}\left(1 - \frac{c}{Kfb}\right) \qquad (10)$$

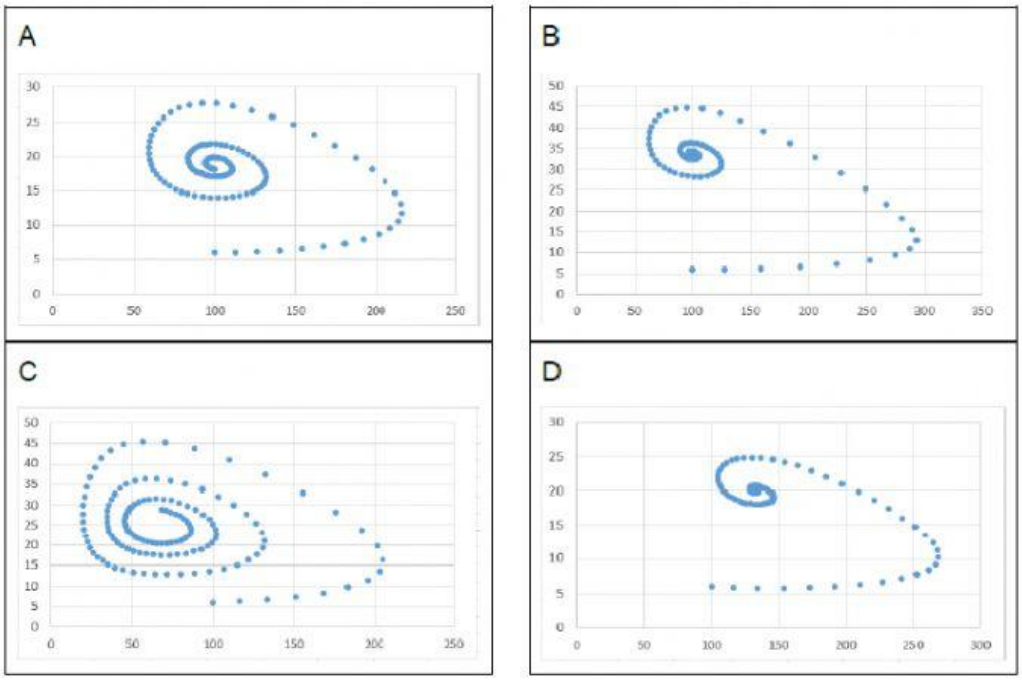
(c) Hence or otherwise, determine the carrying capacity, K. (5)

5.2 A phase plot of a predator-prey model is given below:



LIVEWORKSHEETS

The following phase plots represent the same model with the value of the parameters adjusted.



State which of the graphs represents:

(a) an increase in the parameter '*a*' only. (2)

(b) an increase in the parameter '*f*' only. (2)

[21]

LIVEWORKSHEETS

QUESTION 6

Consider a second-order difference equation of the form:

$u_n = au_{n-1} + bu_{n-2}$

The explicit formula for this sequence can be of the form:

$u_n = Ap^n + Bq^n$

where p and q are the roots of the quadratic equation:

$x^2 - ax + b = 0$

The constants A and B can then be solved using the initial values u_0 and u_1.

6.1 Determine the explicit formula related to the difference equation:

$u_n = 8u_{n-1} + 12u_{n-2}; \quad u_0 = -1; \quad u_1 = 6$ (7)

6.2 Determine the difference equation related to:

$u_n = 3 \times 2^n + 5 \times 3^n$ (5)

LIVEWORKSHEETS