

GRADE 12 EXAMINATION

QUESTION 1

1.1 Solve for $x \in \mathbb{R}$ without using a calculator and showing all working:

(a) $2|e^x - 5| + 3 = 11$ (7)

(b) $\ln x = 3$ (2)

1.2 Determine a and b if $\frac{a+bi}{5-i} = \frac{1}{2} + \frac{1}{2}i$. (10)

- 1.3 Determine, in standard form, a quartic (degree 4) equation with rational coefficients where two of the roots are equal to $2+i$ and $1-\sqrt{3}$. (8)
[27]

QUESTION 2

For a given annual interest rate, the yield is improved by compounding the interest more frequently. However, a limit exists. If interest is compounded continuously then the following formula applies:

$$A = Pe^{rt}$$

Where:

- P is the principle invested
- A is the accumulated amount
- r is the annual interest rate expressed as a percentage
- t is the time in years

- 2.1 By first making t the subject of the formula, determine how long it will take the money invested to triple in value if interest is 10% per annum. Express your answer to the nearest year. (6)

- 2.2 By first making r the subject of the formula, determine the annual interest rate (expressed as a percentage to 2 decimal places) that will increase R500 to a total of R900 in 3 years.

(4)
[10]

QUESTION 3

Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3 for $n \in \mathbb{N}$.

[12]

QUESTION 4

Determine $f'(x)$ by first principles if $f(x) = \sqrt{1-x}$.

[10]

QUESTION 5

5.1 Consider the function $f(x) = \frac{2x^2 + 2x - 3}{x^2 - 5x - 6}$.

(a) Give the equations and nature of all asymptotes.

(6)

(b) Prove that the function is strictly decreasing.

(11)

5.2 Give the equation of a rational function which has:

- an oblique asymptote of $y = 2x + 1$

- a vertical asymptote of $x = -2$
- no x-intercepts