

Misalkan $y = \frac{1}{\sqrt{x}}$, berarti untuk $x \rightarrow \infty$, maka $y \rightarrow 0$. Ekspresi limit berubah menjadi:

$$\lim_{x \rightarrow \infty} x \left(1 - \cos \frac{1}{\sqrt{x}} \right)$$

$$= \lim_{x \rightarrow \infty} (\sqrt{x})^2 \left(1 - \cos \left(\frac{1}{\sqrt{x}} \right) \right)$$

$$= \lim_{y \rightarrow 0} \left(\frac{1}{y^2} \right) (1 - \cos(y))$$

Substitusi permisalan

$$= \lim_{y \rightarrow 0} \left(\frac{1}{y^2} \right) 2 \sin \left(\frac{y}{2} \right) \sin \left(\frac{y}{2} \right)$$

Ubah cos A dengan rumus dasar

$$= 2 \cdot \lim_{y \rightarrow 0} \left(\frac{1}{y} \right) \sin \left(\frac{y}{2} \right) \cdot \lim_{y \rightarrow 0} \left(\frac{1}{y} \right) \sin \left(\frac{y}{2} \right)$$

$$= 2 \cdot \lim_{y \rightarrow 0} \frac{\sin \left(\frac{y}{2} \right)}{\frac{y}{2}} \cdot \lim_{y \rightarrow 0} \frac{\sin \left(\frac{y}{2} \right)}{\frac{y}{2}}$$

Rumus limit trigonometri

$$= 2 \cdot \frac{\sin \left(\frac{y}{2} \right)}{\frac{y}{2}} \cdot \frac{\sin \left(\frac{y}{2} \right)}{\frac{y}{2}}$$

$$= \frac{\sin \left(\frac{y}{2} \right)}{\frac{y}{2}}$$