2 Simplify:

a $3\sqrt{3} \times 2\sqrt{2}$ b $2\sqrt{3} \times 3\sqrt{5}$ c $\sqrt{2} \times \sqrt{3} \times \sqrt{5}$

d $\sqrt{3} \times \sqrt{2} \times 2\sqrt{2}$ e $-3\sqrt{2} \times (\sqrt{2})^2$ f $(3\sqrt{2})^2 \times (\sqrt{3})^2$

3 Simplify:

a $\frac{\sqrt{8}}{\sqrt{2}}$ b $\frac{\sqrt{2}}{\sqrt{8}}$ c $\frac{\sqrt{18}}{\sqrt{2}}$ d $\frac{\sqrt{2}}{\sqrt{18}}$ e $\frac{\sqrt{20}}{\sqrt{5}}$ f $\frac{\sqrt{5}}{\sqrt{20}}$

g $\frac{\sqrt{27}}{\sqrt{3}}$ h $\frac{\sqrt{18}}{\sqrt{3}}$ i $\frac{\sqrt{5}}{\sqrt{30}}$ j $\frac{\sqrt{50}}{\sqrt{2}}$ k $\frac{2\sqrt{6}}{\sqrt{24}}$ l $\frac{5\sqrt{75}}{\sqrt{3}}$

4 Write the following in the form $k\sqrt{2}$:

a $\sqrt{8}$ b $\sqrt{18}$ c $\sqrt{50}$ d $\sqrt{98}$

e $\sqrt{288}$ f $\sqrt{2048}$ g $\sqrt{2010400}$ h $\sqrt{\frac{1}{2}}$

5 Write the following in the form $k\sqrt{3}$:

a $\sqrt{12}$ b $\sqrt{27}$ c $\sqrt{75}$ d $\sqrt{\frac{1}{3}}$

6 Write the following in the form $k\sqrt{5}$:

a $\sqrt{20}$ b $\sqrt{45}$ c $\sqrt{125}$ d $\sqrt{\frac{1}{5}}$

7 a Find:

i $\sqrt{16} + \sqrt{9}$ ii $\sqrt{16+9}$ iii $\sqrt{25} - \sqrt{9}$ iv $\sqrt{25-9}$

b Copy and complete: In general, $\sqrt{a+b} \neq$ ..... and $\sqrt{a-b} \neq$ .....

8 Write the following in simplest surd form:

a $\sqrt{24}$ b $\sqrt{50}$ c $\sqrt{54}$ d $\sqrt{40}$ e $\sqrt{56}$ f $\sqrt{63}$

g $\sqrt{52}$ h $\sqrt{44}$ i $\sqrt{60}$ j $\sqrt{90}$ k $\sqrt{96}$ l $\sqrt{68}$

m $\sqrt{175}$ n $\sqrt{162}$ o $\sqrt{128}$ p $\sqrt{700}$

9 Write the following in simplest surd form:

a $\sqrt{\frac{1}{8}}$ b $\sqrt{\frac{18}{49}}$ c $\sqrt{\frac{44}{81}}$ d $\sqrt{\frac{99}{98}}$

G MULTIPLICATION OF SURDS [1.10]

The rules for expanding brackets involving surds are identical to those for ordinary algebra.

We can thus use:

$$a(b+c) = ab + ac$$
$$(a+b)(c+d) = ac + ad + bc + bd$$
$$(a+b)^2 = a^2 + 2ab + b^2$$
$$(a-b)^2 = a^2 - 2ab + b^2$$
$$(a+b)(a-b) = a^2 - b^2$$

LIVEWORKSHEETS

Example 22 — Self Tutor

Expand and simplify:

a $\sqrt{2}(\sqrt{2}+\sqrt{3})$ b $\sqrt{3}(6-2\sqrt{3})$

a
$$\begin{aligned}&\sqrt{2}(\sqrt{2}+\sqrt{3})\\&=\sqrt{2}\times\sqrt{2}+\sqrt{2}\times\sqrt{3}\\&=2+\sqrt{6}\end{aligned}$$

b
$$\begin{aligned}&\sqrt{3}(6-2\sqrt{3})\\&=(\sqrt{3})(6+-2\sqrt{3})\\&=(\sqrt{3})(6)+(\sqrt{3})(-2\sqrt{3})\\&=6\sqrt{3}+-6\\&=6\sqrt{3}-6\end{aligned}$$

Example 23 — Self Tutor

Expand and simplify:

a $-\sqrt{2}(\sqrt{2}+3)$ b $-\sqrt{3}(7-2\sqrt{3})$

a
$$\begin{aligned}&-\sqrt{2}(\sqrt{2}+3)\\&=-\sqrt{2}\times\sqrt{2}+-\sqrt{2}\times3\\&=-2-3\sqrt{2}\end{aligned}$$

b
$$\begin{aligned}&-\sqrt{3}(7-2\sqrt{3})\\&=(-\sqrt{3})(7-2\sqrt{3})\\&=(-\sqrt{3})(7)+(-\sqrt{3})(-2\sqrt{3})\\&=-7\sqrt{3}+6\end{aligned}$$

Example 24 — Self Tutor

Expand and simplify: $(3-\sqrt{2})(4+2\sqrt{2})$

$$\begin{aligned}&(3-\sqrt{2})(4+2\sqrt{2})\\&=(3-\sqrt{2})(4)+(3-\sqrt{2})(2\sqrt{2})\\&=12-4\sqrt{2}+6\sqrt{2}-4\\&=8+2\sqrt{2}\end{aligned}$$

Example 25 — Self Tutor

Expand and simplify:

a $(\sqrt{3}+2)^2$ b $(\sqrt{3}-\sqrt{7})^2$

a
$$\begin{aligned}&(\sqrt{3}+2)^2\\&=(\sqrt{3})^2+2\times\sqrt{3}\times2+2^2\\&=3+4\sqrt{3}+4\\&=7+4\sqrt{3}\end{aligned}$$

b
$$\begin{aligned}&(\sqrt{3}-\sqrt{7})^2\\&=(\sqrt{3})^2-2\times\sqrt{3}\times\sqrt{7}+(\sqrt{7})^2\\&=3-2\sqrt{21}+7\\&=10-2\sqrt{21}\end{aligned}$$

LIVEWORKSHEETS

Example 26 — Self Tutor

Expand and simplify:

a $(3+\sqrt{2})(3-\sqrt{2})$ b $(2\sqrt{3}-5)(2\sqrt{3}+5)$

a
$$\begin{aligned}&(3+\sqrt{2})(3-\sqrt{2})\\&=3^2-(\sqrt{2})^2\\&=9-2\\&=7\end{aligned}$$

b
$$\begin{aligned}&(2\sqrt{3}-5)(2\sqrt{3}+5)\\&=(2\sqrt{3})^2-5^2\\&=(4\times 3)-25\\&=12-25\\&=-13\end{aligned}$$

EXERCISE 6G

1 Expand and simplify:

a $\sqrt{2}(\sqrt{5}+\sqrt{2})$ b $\sqrt{2}(3-\sqrt{2})$ c $\sqrt{3}(\sqrt{3}+1)$ d $\sqrt{3}(1-\sqrt{3})$

e $\sqrt{7}(7-\sqrt{7})$ f $\sqrt{5}(2-\sqrt{5})$ g $\sqrt{11}(2\sqrt{11}-1)$ h $\sqrt{6}(1-2\sqrt{6})$

i $\sqrt{3}(\sqrt{3}+\sqrt{2}-1)$ j $2\sqrt{3}(\sqrt{3}-\sqrt{5})$ k $2\sqrt{5}(3-\sqrt{5})$ l $3\sqrt{5}(2\sqrt{5}+\sqrt{2})$

2 Expand and simplify:

a $-\sqrt{2}(3-\sqrt{2})$ b $-\sqrt{2}(\sqrt{2}+\sqrt{3})$ c $-\sqrt{2}(4-\sqrt{2})$ d $-\sqrt{3}(1+\sqrt{3})$

e $-\sqrt{3}(\sqrt{3}+2)$ f $-\sqrt{5}(2+\sqrt{5})$ g $-(\sqrt{2}+3)$ h $-\sqrt{5}(\sqrt{5}-4)$

i $-(3-\sqrt{7})$ j $-\sqrt{11}(2-\sqrt{11})$ k $-(\sqrt{3}-\sqrt{7})$ l $-2\sqrt{2}(1-\sqrt{2})$

m $-3\sqrt{3}(5-\sqrt{3})$ n $-7\sqrt{2}(\sqrt{2}+\sqrt{6})$ o $(-\sqrt{2})^2(3-\sqrt{2})$

3 Expand and simplify:

a $(1+\sqrt{2})(2+\sqrt{2})$ b $(2+\sqrt{3})(2+\sqrt{3})$ c $(\sqrt{3}+2)(\sqrt{3}-1)$

d $(4-\sqrt{2})(3+\sqrt{2})$ e $(1+\sqrt{3})(1-\sqrt{3})$ f $(5+\sqrt{7})(2-\sqrt{7})$

g $(\sqrt{5}+2)(\sqrt{5}-3)$ h $(\sqrt{7}-\sqrt{3})(\sqrt{7}+\sqrt{3})$ i $(2\sqrt{2}+\sqrt{3})(2\sqrt{2}-\sqrt{3})$

j $(4-\sqrt{2})(3-\sqrt{2})$

4 Expand and simplify:

a $(1+\sqrt{2})^2$ b $(2-\sqrt{3})^2$ c $(\sqrt{3}+2)^2$ d $(1+\sqrt{5})^2$

e $(\sqrt{2}-\sqrt{3})^2$ f $(5-\sqrt{2})^2$ g $(\sqrt{2}+\sqrt{7})^2$ h $(4-\sqrt{6})^2$

i $(\sqrt{6}-\sqrt{2})^2$ j $(\sqrt{5}+2\sqrt{2})^2$ k $(\sqrt{5}-2\sqrt{2})^2$ l $(6+\sqrt{8})^2$

m $(5\sqrt{2}-1)^2$ n $(3-2\sqrt{2})^2$ o $(1+3\sqrt{2})^2$

5 Expand and simplify:

a $(4+\sqrt{3})(4-\sqrt{3})$ b $(5-\sqrt{2})(5+\sqrt{2})$ c $(\sqrt{5}-2)(\sqrt{5}+2)$

d $(\sqrt{7}+4)(\sqrt{7}-4)$ e $(3\sqrt{2}+2)(3\sqrt{2}-2)$ f $(2\sqrt{5}-1)(2\sqrt{5}+1)$

g $(5-3\sqrt{3})(5+3\sqrt{3})$ h $(2-4\sqrt{2})(2+4\sqrt{2})$ i $(1+5\sqrt{7})(1-5\sqrt{7})$

6 Expand and simplify:

a $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$ b $(\sqrt{7}+\sqrt{11})(\sqrt{7}-\sqrt{11})$ c $(\sqrt{x}-\sqrt{6})(\sqrt{6}+\sqrt{x})$

LIVEWORKSHEETS

H DIVISION BY SURDS [1.10]

When an expression involves division by a surd, we can write the expression with an **integer denominator** which does **not** contain surds.

If the denominator contains a simple surd such as $\sqrt{a}$ then we use the rule $\sqrt{a} \times \sqrt{a} = a$.

For example: $\frac{6}{\sqrt{3}}$ can be written as $\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ since we are really just multiplying the original fraction by 1.

$\frac{6}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ then simplifies to $\frac{6\sqrt{3}}{3}$ or $2\sqrt{3}$.

Example 27 Self Tutor

Express with integer denominator: **a** $\frac{7}{\sqrt{3}}$ **b** $\frac{10}{\sqrt{5}}$ **c** $\frac{10}{2\sqrt{2}}$

a
$$\frac{7}{\sqrt{3}} = \frac{7}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{7\sqrt{3}}{3}$$

b
$$\frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10}{5}\sqrt{5} = 2\sqrt{5}$$

c
$$\frac{10}{2\sqrt{2}} = \frac{10}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{4} = \frac{5\sqrt{2}}{2}$$

If the denominator has the form $a + \sqrt{b}$ then we can remove the surd from the denominator by multiplying both the numerator and the denominator by its **radical conjugate** $a - \sqrt{b}$. This produces a rational denominator, so the process is called **rationalisation** of the denominator.

Example 28 Self Tutor

Express $\frac{1}{3+\sqrt{2}}$ with integer denominator.

$$\frac{1}{3+\sqrt{2}} = \left(\frac{1}{3+\sqrt{2}}\right)\left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right)$$
$$= \frac{3-\sqrt{2}}{3^2-(\sqrt{2})^2} \quad \{\text{using } (a+b)(a-b) = a^2 - b^2\}$$
$$= \frac{3-\sqrt{2}}{7}$$

LIVEWORKSHEETS

Example 29 Self Tutor

Write $\frac{1-2\sqrt{3}}{1+\sqrt{3}}$ in simplest form.

$$\frac{1-2\sqrt{3}}{1+\sqrt{3}} = \left(\frac{1-2\sqrt{3}}{1+\sqrt{3}}\right)\left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right)$$
$$= \frac{1-\sqrt{3}-2\sqrt{3}+6}{1-3}$$
$$= \frac{7-3\sqrt{3}}{-2}$$
$$= \frac{3\sqrt{3}-7}{2}$$

EXERCISE 6H

1 Express with integer denominator:

a $\frac{1}{\sqrt{2}}$ b $\frac{2}{\sqrt{2}}$ c $\frac{4}{\sqrt{2}}$ d $\frac{10}{\sqrt{2}}$ e $\frac{\sqrt{7}}{\sqrt{2}}$

f $\frac{1}{\sqrt{3}}$ g $\frac{3}{\sqrt{3}}$ h $\frac{4}{\sqrt{3}}$ i $\frac{18}{\sqrt{3}}$ j $\frac{\sqrt{11}}{\sqrt{3}}$

k $\frac{1}{\sqrt{5}}$ l $\frac{3}{\sqrt{5}}$ m $\frac{\sqrt{3}}{\sqrt{5}}$ n $\frac{15}{\sqrt{5}}$ o $\frac{125}{\sqrt{5}}$

p $\frac{\sqrt{10}}{\sqrt{2}}$ q $\frac{1}{2\sqrt{3}}$ r $\frac{2\sqrt{2}}{\sqrt{3}}$ s $\frac{15}{2\sqrt{5}}$ t $\frac{1}{(\sqrt{2})^3}$

2 Rationalise the denominator:

a $\frac{1}{3-\sqrt{5}}$ b $\frac{1}{2+\sqrt{3}}$ c $\frac{1}{4-\sqrt{11}}$ d $\frac{\sqrt{2}}{5+\sqrt{2}}$

e $\frac{\sqrt{3}}{3+\sqrt{3}}$ f $\frac{5}{2-3\sqrt{2}}$ g $\frac{-\sqrt{5}}{3+2\sqrt{5}}$ h $\frac{3-\sqrt{7}}{2+\sqrt{7}}$

3 Write in the form $a+b\sqrt{2}$ where $a, b \in \mathbb{Q}$:

a $\frac{4}{2-\sqrt{2}}$ b $\frac{-5}{1+\sqrt{2}}$ c $\frac{1-\sqrt{2}}{1+\sqrt{2}}$ d $\frac{\sqrt{2}-2}{3-\sqrt{2}}$

e $\frac{1+\sqrt{2}}{1-\sqrt{2}}$ f $\frac{1+\sqrt{2}}{1-\sqrt{2}}$ g $\frac{1}{1-\sqrt{2}}$ h $\frac{\sqrt{2}+1}{1-\sqrt{2}}$

Review set 6A

1 Find the integer equal to: a 3^4 b 5×2^3

2 Write as a product of primes in index form: a 36 b 242

LIVEWORKSHEETS

6 Express the following in simplest form, without brackets:

a $(2b^4)^3$ **b** $\left(\frac{3}{x^2y}\right)^2$ **c** $(5a^4b)^2$ **d** $\left(\frac{m^3}{2n^2}\right)^4$

e $\left(\frac{3a^3}{b^5}\right)^3$ **f** $(2m^3n^2)^5$ **g** $\left(\frac{4a^4}{b^2}\right)^2$ **h** $(5x^2y^3)^3$

C ZERO AND NEGATIVE INDICES [1.9, 2.4]

Consider $\frac{2^3}{2^3}$ which is obviously 1.

Using the exponent law for division, $\frac{2^3}{2^3} = 2^{3-3} = 2^0$

We therefore conclude that $2^0 = 1$.

In general, we can state the **zero index law**: $a^0 = 1$ for all $a \neq 0$.

Now consider $\frac{2^4}{2^7}$ which is $\frac{\cancel{2\times2\times2\times2}^{1}}{2\times2\times2\times\cancel{2\times2\times2\times2}_{1}} = \frac{1}{2^3}$

Using the exponent law of division, $\frac{2^4}{2^7} = 2^{4-7} = 2^{-3}$

Consequently, $2^{-3} = \frac{1}{2^3}$, which means that 2^{-3} and 2^3 are **reciprocals** of each other.

In general, we can state the **negative index law**:

> If a is any non-zero number and n is an integer, then $a^{-n} = \frac{1}{a^n}$.
>
> This means that a^n and a^{-n} are **reciprocals** of one another.
>
> In particular notice that $a^{-1} = \frac{1}{a}$.

Using the negative index law, $\left(\frac{2}{3}\right)^{-4} = \frac{1}{\left(\frac{2}{3}\right)^4}$

$= 1 \div \frac{2^4}{3^4}$

$= 1 \times \frac{3^4}{2^4}$

$= \left(\frac{3}{2}\right)^4$

So, in general we can see that: $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ provided $a \neq 0$, $b \neq 0$.

LIVEWORKSHEETS

Example 10 — Self Tutor

Simplify, giving answers in simplest rational form:

a 7^0 **b** 3^{-2} **c** $3^0 - 3^{-1}$ **d** $\left(\frac{5}{3}\right)^{-2}$

a $7^0 = 1$

b $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

c $3^0 - 3^{-1} = 1 - \frac{1}{3} = \frac{2}{3}$

d $\left(\frac{5}{3}\right)^{-2} = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$

EXERCISE 6C

1 Simplify, giving answers in simplest rational form:

a 3^0 **b** 6^{-1} **c** 4^{-1} **d** 5^0

e 3^2 **f** 3^{-2} **g** 5^3 **h** 5^{-3}

i 7^2 **j** 7^{-2} **k** 10^3 **l** 10^{-3}

2 Simplify, giving answers in simplest rational form:

a $(\frac{1}{2})^0$ **b** $\frac{5^4}{5^4}$ **c** $2t^0$ **d** $(2t)^0$

e 7^0 **f** 3×4^0 **g** $\frac{5^3}{5^5}$ **h** $\frac{2^6}{2^{10}}$

i $(\frac{1}{4})^{-1}$ **j** $(\frac{3}{8})^{-1}$ **k** $(\frac{2}{3})^{-1}$ **l** $(\frac{1}{5})^{-1}$

m $2^0 + 2^1$ **n** $5^0 - 5^{-1}$ **o** $3^0 + 3^1 - 3^{-1}$ **p** $(\frac{1}{3})^{-2}$

q $(\frac{2}{3})^{-3}$ **r** $(1\frac{1}{2})^{-3}$ **s** $(\frac{4}{5})^{-2}$ **t** $(2\frac{1}{2})^{-2}$

3 Write the following without brackets or negative indices:

a $(3b)^{-1}$ **b** $3b^{-1}$ **c** $7a^{-1}$ **d** $(7a)^{-1}$

e $\left(\frac{1}{t}\right)^{-2}$ **f** $(4t)^{-2}$ **g** $(5t)^{-2}$ **h** $(5t^{-2})^{-1}$

i xy^{-1} **j** $(xy)^{-1}$ **k** xy^{-3} **l** $(xy)^{-3}$

m $(3pq)^{-1}$ **n** $3(pq)^{-1}$ **o** $3pq^{-1}$ **p** $\frac{(xy)^3}{y^{-2}}$

4 Write as powers of 2, 3 or 5:

a 25 **b** $\frac{1}{25}$ **c** 27 **d** $\frac{1}{27}$

e 16 **f** $\frac{1}{16}$ **g** $\frac{2}{3}$ **h** $\frac{3}{5}$

i $\frac{9}{125}$ **j** $\frac{32}{81}$ **k** $2\frac{2}{3}$ **l** $9\frac{3}{8}$

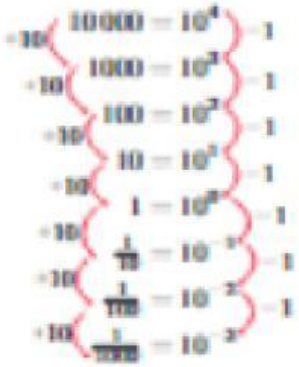
5 Write as powers of 10:

a 1000 **b** 0.01 **c** 100000 **d** 0.000001

LIVEWORKSHEETS

D STANDARD FORM [1.9]

Consider the pattern alongside. Notice that each time we divide by 10, the **exponent** or **power** of 10 decreases by one.



We can use this pattern to simplify the writing of very large and very small numbers.

For example, $5\,000\,000 = 5 \times 1\,000\,000 = 5 \times 10^6$

and $0.000\,003 = \frac{3}{1\,000\,000} = \frac{3}{1} \times \frac{1}{1\,000\,000} = 3 \times 10^{-6}$

STANDARD FORM

Standard form (or **scientific notation**) involves writing any *given number* as *a number between 1 and 10*, multiplied by an *integer power of 10*,

i.e., $a \times 10^n$ where $1 \leqslant a < 10$ and $n \in \mathbb{Z}$.

Example 11 — Self Tutor

Write in standard form: a 57 600 b 0.000 86

a $57\,600 = 5.76 \times 10\,000$ {shift decimal point 4 places to the left and $\times 10\,000$}
$= 5.76 \times 10^4$

b $0.000\,86 = 8.6 \div 10^4$ {shift decimal point 4 places to the right and $\div 10\,000$}
$= 8.6 \times 10^{-4}$

Example 12 — Self Tutor

Write as an ordinary number:

a 3.2×10^2 b 5.76×10^{-5}

a 3.2×10^2
$= 3.20 \times 100$
$= 320$

b 5.76×10^{-5}
$= 000\,005.76 \div 10^5$
$= 0.000\,057\,6$

LIVEWORKSHEETS

Example 13 — Self Tutor

Simplify the following, giving your answer in standard form:

a $(5 \times 10^4) \times (4 \times 10^5)$ b $(8 \times 10^5) \div (2 \times 10^3)$

a
$$\begin{aligned}&(5 \times 10^4) \times (4 \times 10^5)\\ &= 5 \times 4 \times 10^4 \times 10^5\\ &= 20 \times 10^{4+5}\\ &= 2 \times 10^1 \times 10^9\\ &= 2 \times 10^{10}\end{aligned}$$

b
$$\begin{aligned}&(8 \times 10^5) \div (2 \times 10^3)\\ &= \frac{8 \times 10^5}{2 \times 10^3}\\ &= \frac{8}{2} \times 10^{5-3}\\ &= 4 \times 10^2\end{aligned}$$

To help write numbers in standard form:

- If the original number is > 10, the power of 10 is **positive** ($+$).
- If the original number is < 1, the power of 10 is **negative** ($-$).
- If the original number is between 1 and 10, leave it as it is and multiply it by 10^0.

EXERCISE 6D.1

1 Write the following as powers of 10:

a 100 b 1000 c 10 d 100000
e 0.1 f 0.01 g 0.0001 h 100000000

2 Express the following in standard form:

a 387 b 38 700 c 3.87 d 0.0387
e 0.003 87 f 20.5 g 205 h 0.205
i 20 500 j 20 500 000 k 0.000 205

3 Express the following in standard form:

a The circumference of the Earth is approximately 40 075 kilometres.
b The distance from the Earth to the Sun is 149 600 000 000 m.
c Bacteria are single cell organisms, some of which have a diameter of 0.0004 mm.
d There are typically 40 million bacteria in a gram of soil.
e The probability that your six numbers will be selected for Lotto on Saturday night is 0.000 000 141 62.
f Superfine sheep have wool fibres as low as 0.01 mm in diameter.

4 Write as an ordinary decimal number:

a 3×10^2 b 2×10^3 c 3.6×10^4 d 9.2×10^5
e 5.6×10^6 f 3.4×10^1 g 7.85×10^6 h 9×10^8

5 Write as an ordinary decimal number:

a 3×10^{-2} b 2×10^{-3} c 4.7×10^{-4} d 6.3×10^{-5}
e 1.7×10^0 f 9.5×10^{-6} g 3.49×10^{-1} h 7×10^{-8}

LIVEWORKSHEETS

6 Write as an ordinary decimal number:

a The wavelength of visible light is 9×10^{-7} m.

b In 2007, the world population was approximately 6.608×10^{9}.

c The diameter of our galaxy, the Milky Way, is 1×10^{5} light years.

d The smallest viruses are 1×10^{-5} mm in size.

e 1 atomic mass unit is approximately 1.66×10^{-27} kg.

7 Write in standard form:

a 18.17×10^{8} b 0.934×10^{11} c 0.041×10^{-2}

8 Simplify the following, giving your answer in standard form:

a $(8 \times 10^{3}) \times (2 \times 10^{4})$ b $(8 \times 10^{3}) \times (4 \times 10^{5})$

c $(5 \times 10^{6}) \times (3 \times 10^{5})$ d $(2 \times 10^{5})^{3}$

e $(6 \times 10^{2})^{2}$ f $(7 \times 10^{-2})^{2}$

g $(9 \times 10^{6}) \div (3 \times 10^{2})$ h $(8 \times 10^{5}) \div (4 \times 10^{6})$

STANDARD FORM ON A CALCULATOR

Scientific and graphics calculators are able to display very large and very small numbers in standard form.

If you perform 2300000×4000000 your calculator might display 9.2E+11 or 9.2^{11} or 9.2E11, all of which actually represent 9.2×10^{11}.

Likewise, if you perform $0.0024 \div 10000000$ your calculator might display 2.4^{-10} or 2.4E-10, which actually represent 2.4×10^{-10}.

You will find instructions for graphics calculators on page: 16.

EXERCISE 6D.2

1 Write each of the following as it would appear on the display of *your* calculator:

a 4650000 b 0.0000512 c 5.99×10^{-6}

d 3.761×10^{10} e 49500000 f 0.000008844

2 Calculate each of the following, giving your answers in standard form. The decimal part should be correct to 2 decimal places:

a $0.06 \times 0.002 \div 4000$ b $426 \times 780 \times 42000$ c 627000×74000

d $320 \times 600 \times 51400$ e $0.004\,28 \div 120000$ f $0.1026 \times 0.0042 \times 0.188$

Example 14 — Self Tutor

Use your calculator to find:

a $(1.42 \times 10^{6}) \times (2.56 \times 10^{6})$ b $(4.75 \times 10^{-4}) \div (2.5 \times 10^{7})$

Instructions are given for the **Casio fx-6890G**:

Answer:

a 1.42 EXP 6 × 2.56 EXP 8 EXE — 3.6352×10^{12}

b 4.75 EXP (−) 4 ÷ 2.5 EXP 7 EXE — 1.9×10^{-11}

LIVEWORKSHEETS

3 Find, in standard form, with decimal part correct to 2 places:

a $(5.31 \times 10^4) \times (4.8 \times 10^2)$ **b** $(2.75 \times 10^{-2})^2$ **c** $\dfrac{8.24 \times 10^{-5}}{3 \times 10^4}$

d $(7.2 \times 10^{-5}) \div (2.4 \times 10^{-9})$ **e** $\dfrac{1}{4.1 \times 10^7}$ **f** $(3.2 \times 10^5)^2$

4 For the following give answers in standard form correct to 3 significant figures:

a How many millimetres are there in 4738.8 kilometres?

b How many seconds are there in one year?

c How many seconds are there in a millennium?

d How many kilograms are there in 0.5 milligrams?

5 If a missile travels at 3600 km/h, how far will it travel in:

a 1 day **b** 1 week **c** 2 years?

Give your answers in standard form with decimal part correct to 2 places. Assume that 1 year = 365 days.

6 Light travels at a speed of 3×10^8 metres per second. How far will light travel in:

a 1 minute **b** 1 day **c** 1 year?

Give your answers in standard form with decimal part correct to 2 decimal places. Assume that 1 year = 365 days.

E SURDS [1.10]

For the remainder of this chapter we consider **surds** and **radicals**, which are numbers that are written using the **radical** or **square root sign** $\sqrt{}$.

Surds and radicals occur frequently in mathematics, often as solutions to equations involving squared terms. We will see a typical example of this in **Chapter 8** when we study Pythagoras' theorem.

RATIONAL AND IRRATIONAL RADICALS

Some radicals are rational, but most are irrational.

For example, some rational radicals include: $\sqrt{1} = \sqrt{1^2} = 1$ or $\frac{1}{1}$

$\sqrt{4} = \sqrt{2^2} = 2$ or $\frac{2}{1}$

$\sqrt{\frac{1}{4}} = \sqrt{\left(\frac{1}{2}\right)^2} = \frac{1}{2}$

Two examples of irrational radicals are $\sqrt{2} \approx 1.414\,214$

and $\sqrt{3} \approx 1.732\,051$.

Strictly speaking, a **surd** is an **irrational radical**. However, in this and many other courses, the term *surd* is used to describe *any* radical. It is reasonable to do so because the properties of surds and radicals are the same.

LIVEWORKSHEETS

Historical note

The name **surd** and the **radical** sign $\sqrt{}$ both had a rather absurd past. Many centuries after Pythagoras, when the Golden Age of the Greeks was past, the writings of the Greeks were preserved, translated, and extended by Arab mathematicians.

The Arabs thought of a square number as growing out of its roots. The roots had to be extracted. The Latin word for "root" is **radix**, from which we get the words **radical** and radish! The printed symbol for radix was first R, then r, which was copied by hand as $\sqrt{}$.

The word **surd** actually came about because of an error of translation by the Arab mathematician Al-Khwarizmi in the 9th century AD. The Greek word **a-logos** means "irrational" but also means "deaf". So, the Greek **a-logos** was interpreted as "deaf" which in Latin is **surdus**. Hence to this day, **irrational radicals** like $\sqrt{2}$ are called **surds**.

BASIC OPERATIONS WITH SURDS

We have seen square roots and cube roots in previous courses. We can use their properties to help with some simplifications.

Example 15 — Self Tutor

Simplify:

a $(\sqrt{5})^2$ b $\left(\frac{1}{\sqrt{5}}\right)^2$

a
$$(\sqrt{5})^2$$
$$= \sqrt{5} \times \sqrt{5}$$
$$= 5$$

b
$$\left(\frac{1}{\sqrt{5}}\right)^2$$
$$= \frac{1}{\sqrt{5}} \times \frac{1}{\sqrt{5}}$$
$$= \frac{1}{5}$$

Example 16 — Self Tutor

Simplify:

a $(2\sqrt{5})^2$ b $-2\sqrt{5} \times 3\sqrt{5}$

a
$$(2\sqrt{5})^2$$
$$= 2\sqrt{5} \times 2\sqrt{5} \times 2\sqrt{5}$$
$$= 2 \times 2 \times 2 \times \sqrt{5} \times \sqrt{5} \times \sqrt{5}$$
$$= 8 \times 5 \times \sqrt{5}$$
$$= 40\sqrt{5}$$

b
$$-2\sqrt{5} \times 3\sqrt{5}$$
$$= -2 \times 3 \times \sqrt{5} \times \sqrt{5}$$
$$= -6 \times 5$$
$$= -30$$

LIVEWORKSHEETS

ADDING AND SUBTRACTING SURDS

"Like surds" can be added and subtracted in the same way as "like terms" in algebra.

Consider $2\sqrt{3} + 4\sqrt{3}$, which has the same form as $2x + 4x$.

If we interpret this as 2 "lots" of $\sqrt{3}$ plus 4 "lots" of $\sqrt{3}$, we have 6 "lots" of $\sqrt{3}$.

So, $2\sqrt{3} + 4\sqrt{3} = 6\sqrt{3}$, and we can compare this with $2x + 4x = 6x$.

Example 17 — Self Tutor

Simplify:

a $3\sqrt{2} + 4\sqrt{2}$ b $5\sqrt{3} - 6\sqrt{3}$

a $3\sqrt{2} + 4\sqrt{2}$
$= 7\sqrt{2}$
{Compare: $3x + 4x = 7x$}

b $5\sqrt{3} - 6\sqrt{3}$
$= -1\sqrt{3}$
$= -\sqrt{3}$
{Compare: $5x - 6x = -x$}

EXERCISE 6E

1 Simplify:

a $(\sqrt{7})^2$ b $(\sqrt{13})^2$ c $(\sqrt{15})^2$ d $(\sqrt{26})^2$

e $\left(\frac{1}{\sqrt{3}}\right)^2$ f $\left(\frac{1}{\sqrt{11}}\right)^2$ g $\left(\frac{1}{\sqrt{17}}\right)^2$ h $\left(\frac{1}{\sqrt{23}}\right)^2$

2 Simplify:

a $(\sqrt[3]{2})^3$ b $(\sqrt[3]{-5})^3$ c $\left(\frac{1}{\sqrt[3]{25}}\right)^3$

3 Simplify:

a $3\sqrt{2} \times 4\sqrt{2}$ b $-2\sqrt{3} \times 5\sqrt{3}$ c $3\sqrt{5} \times (-2\sqrt{5})$

d $-2\sqrt{2} \times (-3\sqrt{2})$ e $(3\sqrt{2})^2$ f $(3\sqrt{2})^3$

g $(2\sqrt{3})^2$ h $(2\sqrt{3})^3$ i $(2\sqrt{2})^4$

4 Simplify:

a $\sqrt{2} + \sqrt{2}$ b $\sqrt{2} - \sqrt{2}$ c $3\sqrt{2} - 2\sqrt{2}$

d $2\sqrt{3} - \sqrt{3}$ e $5\sqrt{7} + 2\sqrt{7}$ f $3\sqrt{5} - 6\sqrt{5}$

g $3\sqrt{2} + 4\sqrt{2} - \sqrt{2}$ h $6\sqrt{2} - 9\sqrt{2}$ i $\sqrt{5} + 7\sqrt{5}$

j $3\sqrt{2} - 5\sqrt{2} - \sqrt{2}$ k $3\sqrt{3} - \sqrt{3} + 2\sqrt{3}$ l $3\sqrt{5} + 7\sqrt{5} - 10$

5 Simplify:

a $3\sqrt{2} + 2\sqrt{3} - \sqrt{2} + 5\sqrt{3}$ b $7\sqrt{2} - 4\sqrt{3} - 2\sqrt{2} + 3\sqrt{3}$

c $-6\sqrt{2} - 2\sqrt{3} - \sqrt{2} + 6\sqrt{3}$ d $2\sqrt{5} + 4\sqrt{2} + 9\sqrt{5} - 9\sqrt{2}$

e $3\sqrt{2} - 5\sqrt{7} - \sqrt{2} - 5\sqrt{7}$ f $3\sqrt{2} + 4\sqrt{11} + 6 - \sqrt{2} - \sqrt{11} - 3$

g $6\sqrt{6} - 2\sqrt{2} - \sqrt{2} - 5\sqrt{6} + 4$ h $5\sqrt{3} - 6\sqrt{7} - 5 + 4\sqrt{3} + \sqrt{7} - 8$

LIVEWORKSHEETS

F PROPERTIES OF SURDS [1.10]

Discovery — Properties of surds

Notice that $\sqrt{4 \times 9} = \sqrt{36} = 6$ and $\sqrt{4} \times \sqrt{9} = 2 \times 3 = 6$, which suggests that $\sqrt{4} \times \sqrt{9} = \sqrt{4 \times 9}$.

Also, $\sqrt{\frac{36}{4}} = \sqrt{9} = 3$ and $\frac{\sqrt{36}}{\sqrt{4}} = \frac{6}{2} = 3$, which suggests that $\frac{\sqrt{36}}{\sqrt{4}} = \sqrt{\frac{36}{4}}$.

What to do:

Test the following possible properties or rules for surds by substituting different values of a and b. Use your calculator to evaluate the results.

1. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ for all $a \geqslant 0, b \geqslant 0$.
2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ for all $a \geqslant 0, b > 0$.
3. $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ for all $a \geqslant 0, b \geqslant 0$.
4. $\sqrt{a-b} = \sqrt{a} - \sqrt{b}$ for all $a \geqslant 0, b \geqslant 0$.

You should have discovered the following properties of surds:

- $\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$ for $a \geqslant 0$, $b \geqslant 0$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ for $a \geqslant 0$, $b > 0$

However, in general it is not true that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ or that $\sqrt{a-b} = \sqrt{a} - \sqrt{b}$.

Example 18 — Self Tutor

Write in simplest form:

a $\sqrt{3} \times \sqrt{2}$ b $2\sqrt{5} \times 3\sqrt{2}$

a
$$\sqrt{3} \times \sqrt{2} = \sqrt{3 \times 2} = \sqrt{6}$$

b
$$2\sqrt{5} \times 3\sqrt{2} = 2 \times 3 \times \sqrt{5} \times \sqrt{2} = 6 \times \sqrt{5 \times 2} = 6\sqrt{10}$$

LIVEWORKSHEETS

Example 19 — Self Tutor

Simplify: a $\frac{\sqrt{32}}{\sqrt{2}}$ b $\frac{\sqrt{12}}{2\sqrt{3}}$

a $\frac{\sqrt{32}}{\sqrt{2}}$
$= \sqrt{\frac{32}{2}}$
$= \sqrt{16}$
$= 4$

b $\frac{\sqrt{12}}{2\sqrt{3}}$
$= \frac{1}{2}\sqrt{\frac{12}{3}}$ {using $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$}
$= \frac{1}{2}\sqrt{4}$
$= \frac{1}{2} \times 2$
$= 1$

Example 20 — Self Tutor

Write $\sqrt{32}$ in the form $k\sqrt{2}$.

$\sqrt{32}$
$= \sqrt{16 \times 2}$
$= \sqrt{16} \times \sqrt{2}$ {using $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$}
$= 4\sqrt{2}$

SIMPLEST SURD FORM

A surd is in **simplest form** when the number under the radical sign is the smallest integer possible.

Example 21 — Self Tutor

Write $\sqrt{28}$ in simplest surd form.

$\sqrt{28}$
$= \sqrt{4 \times 7}$ {4 is the largest perfect square factor of 28}
$= \sqrt{4} \times \sqrt{7}$
$= 2\sqrt{7}$

EXERCISE 6F

1 Simplify:

a $\sqrt{2} \times \sqrt{5}$ b $\sqrt{3} \times \sqrt{7}$ c $\sqrt{3} \times \sqrt{11}$
d $\sqrt{7} \times \sqrt{7}$ e $\sqrt{3} \times 2\sqrt{3}$ f $2\sqrt{2} \times \sqrt{5}$

LIVEWORKSHEETS