

AXIOMS OF REAL NUMBERS & BINARY SYSTEMS

Name: _____
 Date: _____
 Class: _____

LEARNING OUTCOMES

You should be able to:

- Identify and apply the field axioms of real numbers
- Understand the order axioms of real numbers
- Explain what binary numbers are and how base systems work
- Convert between decimal and binary numbers

PART A – FIELD AXIOMS OF REAL NUMBERS

Fill in the missing words or examples:

1. CLOSURE: If a and b are real numbers, then $a + b$ is a real number and $a \times b$ is a real number.
 Example: 3 and 5 are real numbers. So $3 + 5 = \underline{\hspace{1cm}}$ is also a real number.

2. COMMUTATIVE: Changing the order does not change the result.
 Addition rule: $a + b = \underline{\hspace{1cm}}$
 Example: $7 + 2 = 2 + \underline{\hspace{1cm}}$
 Multiplication rule: $a \times b = \underline{\hspace{1cm}}$
 Example: $4 \times 6 = \underline{\hspace{1cm}} \times 4$

3. ASSOCIATIVE: Changing the grouping does not change the result.
 Addition rule: $(a + b) + c = \underline{\hspace{1cm}}$
 Example: $(2 + 3) + 4 = 2 + (\underline{\hspace{1cm}} + \underline{\hspace{1cm}})$
 Multiplication rule: $(a \times b) \times c = \underline{\hspace{1cm}}$
 Example: $(2 \times 5) \times 3 = 2 \times (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}})$

4. DISTRIBUTIVE: Multiplication over addition.
 Rule: $a(b + c) = \underline{\hspace{1cm}}$
 Example: $3(2 + 4) = (3 \times \underline{\hspace{1cm}}) + (3 \times \underline{\hspace{1cm}}) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

5. IDENTITY ELEMENTS:
 Addition identity: $a + \underline{\hspace{1cm}} = a$
 Example: $9 + 0 = \underline{\hspace{1cm}}$
 Multiplication identity: $a \times \underline{\hspace{1cm}} = a$
 Example: $7 \times 1 = \underline{\hspace{1cm}}$

6. INVERSE ELEMENTS:
 Additive inverse: $a + \underline{\hspace{1cm}} = 0$
 Example: $5 + (\underline{\hspace{1cm}}) = \underline{\hspace{1cm}}$
 Multiplicative inverse: $a \times \underline{\hspace{1cm}} = 1$ (where $a \neq 0$)
 Example: $4 \times \frac{1}{4} = \underline{\hspace{1cm}}$

PART B – IDENTIFY THE AXIOM

Write the name of the axiom illustrated in each example:

- $8 + 3 = 3 + 8$

- $5 \times (2 \times 6) = (5 \times 2) \times 6$

- $4 + 0 = 4$

- $6(3 + 2) = 6 \times 3 + 6 \times 2$

- $7 + (-7) = 0$

- $6 \times 1 = 6$

- If 2 and 5 are real numbers, then 2×5 is also a real number

- $2 \times (1/2) = 1$

PART C – ORDER AXIOMS OF REAL NUMBERS

Fill in the blanks:

- TRICHOTOMY PROPERTY: For any two real numbers a and b , exactly one of the following is true:
 $a < b$, _____, or _____
- TRANSITIVE PROPERTY: If $a < b$ and $b < c$, then _____
 Example: If $3 < 5$ and $5 < 7$, then _____
- ORDER PRESERVATION UNDER ADDITION: If $a < b$, then $a + c$ _____ $b + c$
 Example: Since $2 < 4$, then $2 + 3$ _____ $4 + 3$
- ORDER PRESERVATION UNDER POSITIVE MULTIPLICATION: If $a < b$ and $c > 0$, then $a \times c$ _____ $b \times c$
 Example: Since $2 < 5$ and $3 > 0$, then 2×3 _____ 5×3

PART D – INTRODUCTION TO BINARY SYSTEMS

Answer these questions:

- What is the base of the decimal number system? _____
- What is the base of the binary number system? _____
- Which two digits are used in the binary system? _____ and _____
- What does "positional notation" mean?
- Name two other common number bases besides decimal and binary: _____ and _____

PART E – CONVERTING DECIMAL TO BINARY

Fill in the missing values. Use the place values: 128, 64, 32, 16, 8, 4, 2, 1

Example: $5 = 4 + 1 = 101_2$

- $6 = 4 + 2 = \underline{\hspace{1cm}}_2$
- $9 = 8 + 1 = \underline{\hspace{1cm}}_2$
- $12 = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}_2$
- $18 = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}_2$
- $25 = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}_2$

PART F – CONVERTING BINARY TO DECIMAL

Calculate the decimal value:

Example: $101_2 = (1 \times 4) + (0 \times 2) + (1 \times 1) = 4 + 0 + 1 = 5$

- $11_2 = (1 \times 2) + (1 \times 1) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- $110_2 = (1 \times 4) + (1 \times 2) + (0 \times 1) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- $1001_2 = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- $1101_2 = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- $10110_2 = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$