In this exercise you have:

- used factor trees to write an integer as a product of prime factors
- found the HCF of two integers by first writing each one as a product of prime numbers
- found the LCM of two integers by first writing each one as a product of prime numbers.

a Which questions have you found the easiest? Explain why.

b Which questions have you found the hardest? Explain why.

Summary checklist

☐ I can write an integer as a product of prime numbers.

☐ I can find the HCF and LCM of two integers by first writing each one as a product of prime numbers.

1.2 Multiplying and dividing integers

In this section you will …

- multiply and divide integers, in particular when both are negative
- understand that brackets, indices and operations follow a particular order.

Key words

brackets
conjecture
inverse
investigate

You can add and subtract any two integers.

For example:

$2 + -4 = -2$ $\quad -2 + -4 = -6$ $\quad -2 - 4 = -6$ $\quad -2 - -4 = 2$

You can also multiply and divide a negative integer by a positive one.

For example:

$2 \times -9 = -18$ $\quad -6 \times 3 = -18$ $\quad -18 \div 3 = -6$ $\quad 20 \div -5 = -4$

In this section you will **investigate** how to multiply or divide any two integers. You will use number patterns to do this.

Worked example 1.2

Look at this sequence of subtractions.

$3 - 6 = -3$
$3 - 4 = -1$
$3 - 2 =$
$3 - 0 =$
$3 - -2 =$
$3 - -4 =$

a Copy the sequence and fill in the missing answers.
b Write the next three lines in the sequence.
c Describe any patterns in the sequence.

A sequence is a set of numbers or expressions made and written in order, according to some pattern.

Answer

a $3 - 2 = 1$
$3 - 0 = 3$
$3 - -2 = 5$
$3 - -4 = 7$

b $3 - -6 = 9$
$3 - -8 = 11$
$3 - -10 = 13$

c The first number, 3, does not change.
The number being subtracted decreases by 2 each time.
The answer increases by 2 each time.

Exercise 1.2

Think like a mathematician

1 Here is the start of a sequence of multiplications.
$-3 \times 4 = -12$
$-3 \times 3 =$
$-3 \times 2 =$

a Copy the sequence and write six more terms. Use a pattern to fill in the answers.
b Describe the patterns in the sequence.

Continued

c Here is the start of another sequence of multiplications.

$-5 \times 4 =$

$-5 \times 3 =$

$-5 \times 2 =$

Copy the sequence and write six more terms.
Describe any patterns in the sequence.

d In the sequences in **a** and **c**, you have some products of two negative integers.
What can you say about the product of two negative integers?

e Make up a sequence of your own like the ones in **a** and **c**.

f Share your answers to parts **d** and **e** with a partner. Are your partner's sequences correct?

2 Work out these multiplications.

a 5×-2 **b** -5×2 **c** -5×-2 **d** -2×-5

3 Work out these multiplications.

a -6×-4 **b** -7×-7 **c** -10×-6 **d** -8×-11

4 Copy and complete this multiplication table.

×	−5	3	−8
4			
−3		−9	
−6	30		

5 Work out

a $(3+5) \times -4$ **b** $(-3+-5) \times -6$

c $-4 \times (5-8)$ **d** $-6 \times (-2--7)$

Tip

Do the calculation in **brackets** first.

6 Round these numbers to the nearest whole number to estimate the answer.

a 3.9×-6.8 **b** -11.2×2.95

c $(-6.1)^2$ **d** $(-4.88)^2$

7 **a** Put these multiplications into groups based on the answers.

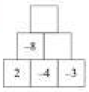
3×-4 -6×-2 12×1

-4×-3 2×-6 -12×-1

b Find one more product to put in each group.

8 These are multiplication pyramids.

a



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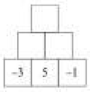

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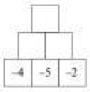
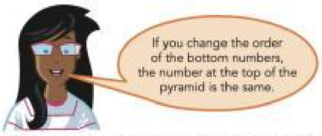


Each number is the product of the two numbers below it. For example, in **a**, $2 \times -4 = -8$

Copy and complete the multiplication pyramids.

9 **a** Draw a multiplication pyramid like those in Question 8, with the integers −2, 3 and −5 in the bottom row, in that order. Complete your pyramid.



b Is Zara correct? Test her idea by changing the order of the numbers in the bottom row of your pyramid.

10 Find the missing numbers in these multiplications.

a $-3 \times \square = -12$ **b** $-5 \times \square = 45$

c $\square \times -6 = 24$ **d** $\square \times -10 = 80$

Think like a mathematician

11 A multiplication can be written as a division.

For example, $5 \times 8 = 40$ can be written as $40 \div 8 = 5$ or $40 \div 5 = 8$

a Here is a multiplication: $-4 \times 6 = -24$

Write it as a division in two different ways.

b Write a multiplication of a positive integer and a negative integer.

Then write it as a division in two different ways.

c Here is a multiplication: $-7 \times -2 = 14$

Write it as a division in two different ways.

d Write a multiplication of two negative integers.

Then write it as a division in two different ways.

Tip

A conjecture is a possible value based on what you know.

Continued

e Can you make a conjecture about the answer when you divide an integer by a negative integer?
Test your conjecture.

f Compare your answer with a partner's answers.
Have you made the same conjectures?

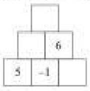
12 Work out these divisions.

a $18 \div -6$ b $-28 \div -4$ c $30 \div -6$

d $-30 \div -10$ e $42 \div -6$ f $-24 \div -4$

g $60 \div -5$ h $-25 \div -5$

13 Here are three multiplication pyramids.

a


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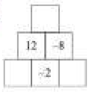

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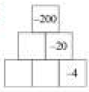


Copy and complete each pyramid.

Tip

Remember, division is the inverse of multiplication so you will divide as you work down the pyramid.

14 Work out

a $(3 \times -4) \div -2$ b $(2 - 20) \div -3$

c $(-3 + 15) \div -4$ d $24 \div (2 \times -4)$

15 Find the value of x.

a $x \div -4 = 8$ b $x \div -3 = -15$

c $16 \div x = -2$ d $-15 \div x = 3$

16 Round these numbers to the nearest whole number to estimate the answer.

a $-8.75 \div 2.8$ b $18.1 \div -5.9$

c $-28.2 \div -3.8$ d $-35.2 \div -6.9$

17 Round these numbers to the nearest 10 to estimate the answer.

a -48×-29 b -18.1×61.5

c $-71.4 \div -11.8$ d $-99.4 \div 19$

Summary checklist

- [ ] I can multiply two negative integers.
- [ ] I can divide any integer by a negative integer.

1.3 Square roots and cube roots

In this section you will ...

- find the squares of positive and negative integers and their corresponding square roots
- find the cubes of positive and negative integers and their corresponding cube roots
- learn to recognise natural numbers, integers and rational numbers.

Key words

cube root
natural numbers
rational numbers
square root

$5^2 = 25$

This means that the **square root** of 25 is 5. This can be written as $\sqrt{25} = 5$.

This is the only answer in the set of **natural numbers**.

Tip

The natural numbers are the counting numbers and zero.

However $(-5)^2 = -5 \times -5 = 25$

This means that the integer -5 is also a square root of 25.

Every positive integer has two square roots, one positive and one negative.

5 is the positive square root of 25 and -5 is the negative square root.

No negative number has a square root.

For example, the integer -25 has no square root because the equation $x^2 = -25$ has no solution.

$5^3 = 125$

This means that the **cube root** of 125 is 5. This can be written as $\sqrt[3]{125} = 5$.

You might think -5 is also a cube root of 125.

However $(-5)^3 = -5 \times -5 \times -5 = (-5 \times -5) \times -5 = 25 \times -5 = -125$

So $\sqrt[3]{-125} = -5$

Every number, positive or negative or zero, has only one cube root.

Worked example 1.3

Solve each equation.

a $x^2 = 64$

b $x^3 = 64$

c $x^3 + 64 = 0$

Continued

Answer

a 64 has two square roots. One is $\sqrt{64} = 8$ and the other is $-\sqrt{64} = -8$
So the equation has two solutions: $x = 8$ or $x = -8$

b $\sqrt[3]{64} = 4$. This means $4^3 = 4 \times 4 \times 4 = 64$ and so $x = 4$

c If $x^3 + 64 = 0$ then $x^3 = -64$. So $x = \sqrt[3]{-64} = -4$

Exercise 1.3

1 Work out

a 7^2 **b** $(-7)^2$ **c** 7^3 **d** $(-7)^3$

2 Find

a $\sqrt[3]{125}$ **b** $\sqrt[3]{-27}$ **c** $\sqrt[3]{-1}$ **d** $\sqrt[3]{-8}$

3 Solve these equations.

a $x^2 = 100$ **b** $x^2 = 144$ **c** $x^2 = 1$
d $x^2 = 0$ **e** $x^2 + 9 = 0$

4 Solve these equations.

a $x^3 = 216$ **b** $x^3 + 27 = 0$
c $x^3 + 1 = 0$ **d** $x^3 + 125 = 0$

5 $27^2 = 9^3 = 729$
Use this fact to find

a $\sqrt{729}$ **b** $\sqrt{-729}$ **c** $\sqrt[3]{729}$ **d** $\sqrt[3]{-729}$

6 **a** A calculator shows that $\sqrt{8^2} - \sqrt{(-8)^2} = 0$
Explain why this is correct.

b Find the value of $\sqrt[3]{4^3} - \sqrt[3]{(-4)^3}$. Show your working.

7 The square of an integer is 100.
What can you say about the cube of the integer?

8 The integer $1521 = 3^2 \times 13^2$
Use this fact to

a find $\sqrt{1521}$

b solve the equation $x^2 = 1521$

9 **a** How is -5^2 different from $(-5)^2$?

b What is the difference between -5^3 and $(-5)^3$?

10 a Show that $3^2 + 4^2 = 5^2$

b Are these statements true or false?
Give a reason for your answer each time.

i $(-3)^2 + (-4)^2 = (-5)^2$

ii $(-13)^2 = 12^2 + (-5)^2$

iii $8^2 = -10^2 - 6^2$

c Show your work to a partner.
Do they find your explanation clear?

Think like a mathematician

11 a Here is an equation: $x^2 + x = 6$

i Show that $x = 2$ is a solution of the equation.

ii Show that $x = -3$ is a solution of the equation.

b Here is another equation: $x^2 + x = 12$

i Show that $x = 3$ is a solution of the equation.

ii Find a second solution to the equation.

c Find two solutions to this equation: $x^2 + x = 20$

d What patterns can you see in the answers to **a**, **b** and **c**?
Find some more equations like this and write down the solutions.

e Compare your answers with a partner's.

12 a Copy and complete this table.

x	$x - 1$	$x^3 - 1$	$x^2 + x + 1$
2	1	7	
3	2		13
4			
5			

b What pattern can you see in your answers?

c Add another row to see if the pattern is still the same.

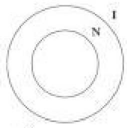
d Add three rows where x is a negative integer.
Is the pattern still the same if x is a negative integer?

e Compare your answers with a partner's.

13 Any number that can be written as a fraction is a **rational number**.
Examples are $7\frac{3}{4}$, $-12\frac{18}{25}$, 6, $\frac{1}{15}$, $-2\frac{9}{10}$
Here is a list of six numbers:
5 $-\frac{1}{5}$ -500 16 -4.8 $99\frac{1}{2}$
Write
- a all the integers in the list
- b all the natural numbers in the list
- c all the rational numbers in the list.

> **Tip**
> Integers and fractions are included in the set of rational numbers.

14 This Venn diagram shows the relationship between natural numbers and integers.
N stands for natural numbers and I for integers.



- a Copy the Venn diagram.
- b Write each of these numbers in the correct part of the diagram.
  1 -3 7 -12 41 -100 $2\frac{1}{2}$
- c Add another circle to your Venn diagram to show rational numbers.
- d Add these numbers to your Venn diagram.
  $-8\frac{3}{7}$ $\frac{3}{5}$ 0 6.3 $-\frac{10}{3}$
- e Give your diagram to a partner to check.

> **Tip**
> Remember, all integers are included in the rational numbers.

Summary checklist

- ☐ I can find and recognise square numbers and their two corresponding square roots.
- ☐ I can find and recognise positive and negative cube numbers and their cube roots.
- ☐ I can recognise natural numbers, integers and rational numbers.

1.4 Indices

In this section you will …

- use positive and zero indices to represent numbers and in multiplication and division.

Key words

generalise
power

In this section you will investigate numbers written as powers.
Look at these powers of 5

n	0	1	2	3	4	5
5^n			25	125	625	3125

So $5^3 = 5 \times 5 \times 5 = 125$ and $5^4 = 5 \times 5 \times 5 \times 5 = 625$ and so on.
As you move to the **right** the numbers in the bottom row **multiply** by 5.
As you move to the **left** the numbers in the bottom row **divide** by 5.
$3125 \div 5 = 625$, $625 \div 5 = 125$, $125 \div 5 = 25$
If you continue to divide by 5, $25 \div 5 = 5$ so $5^1 = 5$
There is another number missing in the table. What is 5^0?
Divide by 5 again: $5^0 = 5^1 \div 5 = 5 \div 5 = 1$
So $5^0 = 1$
If n is any positive integer then $n^0 = 1$.

Worked example 1.4

a Show that $7^3 = 343$
b Work out
  i 7^4
  ii 7^0

Answer

a $7^3 = 7 \times 7 \times 7 = 49 \times 7 = 343$
b **i** $7^4 = 7^3 \times 7 = 343 \times 7 = 2401$
  ii $7^0 = 1$

Exercise 1.4

1 Copy and complete this list of powers of 2.

Power	2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}
Number	1	2		8			64			512	

2 Copy and complete this list of powers of 3.

Power	3^0	3^1	3^2	3^3	3^4	3^5	3^6	3^7	3^8
Number		3		27				2187	

Think like a mathematician

3 Look at this multiplication: $4 \times 16 = 64$

You can write all the numbers as powers of 2: $2^2 \times 2^4 = 2^6$

a Write each of these multiplications as powers of 2.

i $8 \times 4 = 32$ ii $16 \times 8 = 128$

iii $4 \times 32 = 128$ iv $2 \times 128 = 256$

v $16 \times 32 = 512$

b Can you see a pattern in your answers? Make a conjecture about multiplying powers of 2.
Test your conjecture on some more multiplications of your own.

c Make a conjecture about multiplying powers of 3.
Use some examples to test your conjecture.

d Generalise your results so far.

Tip

'Generalising' means using a set of results to come up with a general rule.

4 Write the answers to these calculations as powers of 6.

a $6^2 \times 6^3$ b $6^4 \times 6$ c $6^5 \times 6^2$ d $6^3 \times 6^3$

5 Write the answers to these calculations in index form.

a $10^3 \times 10^2$ b $20^5 \times 20$ c $15^3 \times 15^3$ d $5^3 \times 5^5$

6 a $3^8 = 6561$ Use this fact to find 3^9 and show your method.

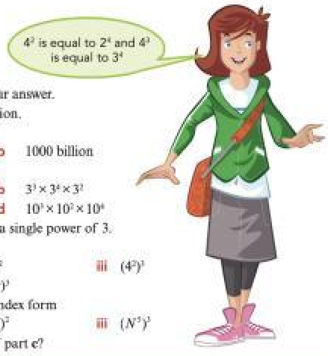
b $5^6 = 15\,625$ Use this fact to find 5^7 and show your method.

7 Find the missing power.

a $3^3 \times 3^{\square} = 3^5$ b $9^3 \times 9^{\square} = 9^8$

c $12^4 \times 12^{\square} = 12^6$ d $15^{\square} \times 15^3 = 15^{10}$

8 Read what Sofia says.



Is Sofia correct? Give a reason for your answer.

9 A million is 10^6. A billion is 1000 million.
Write as a power of 10

a one billion
b 1000 billion

10 Write in index form

a $2^2 \times 2^5 \times 2$
b $3^3 \times 3^4 \times 3^2$
c $5 \times 5^3 \times 5^3$
d $10^3 \times 10^2 \times 10^6$

11 a $(3^2)^3 = 3^2 \times 3^2 \times 3^2$ Write $(3^2)^3$ as a single power of 3.

b Write in index form

i $(2^3)^2$
ii $(5^3)^2$
iii $(4^2)^3$
iv $(15^2)^4$
v $(10^4)^3$

c N is a positive integer. Write in index form

i $(N^2)^3$
ii $(N^4)^2$
iii $(N^5)^2$

d Can you generalise the results of part **c**?

Think like a mathematician

12 Here is a division: $32 \div 4 = 8$
You can write this using indices: $2^5 \div 2^2 = 2^3$

a Write each of these divisions using indices. All the numbers are powers of 2 or 3.

i $64 \div 4 = 16$
ii $81 \div 3 = 27$
iii $512 \div 16 = 32$
iv $729 \div 9 = 81$
v $9 \div 9 = 1$

b Write some similar divisions using powers of 5.

c Can you generalise your results from **a** and **b**?
Check with some powers of other positive integers.

d Compare your results with a partner's.

13 Write the answers to these calculations in index form.

a $2^7 \div 2^3$
b $10^6 \div 10^3$
c $15^8 \div 15^6$
d $8^{10} \div 8^9$
e $2^{15} \div 2^{11}$
f $2^5 \div 2^3$

14 Write the answers to these calculations in index form.

a $9^3 \times 9^2$
b $9^3 \div 9^2$
c $(9^3)^2$
d $5^3 \times 5^4$
e $12^8 \div 12^3$
f $(7^5)^3$
g $(10^5)^4$

15 Read what Zara says.

a Is Zara correct? Give a reason for your answer.

b Is a similar result true for other indices?

16 $15 = 3 \times 5$

Use this fact to write as a product of prime factors

a 15^2 **b** 15^3 **c** 15^5 **d** 15^8

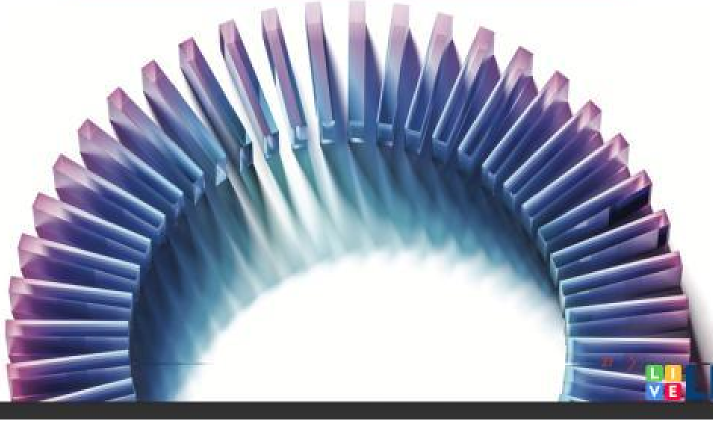
17 **a** Write $5^6 \div 5^4$ as a power of 5.

b Write $5^6 \div 5^6$ as a power of 5.

c Is it possible to write $5^4 \div 5^6$ as a power of 5?

Summary checklist

- ☐ I can use index notation for positive integers where the index is a positive integer or zero.
- ☐ I can multiply and divide numbers written as powers of a positive integer.



Check your progress

1 a Draw a factor tree for 350.
b Write 350 as a product of prime factors.
c Write 112 as a product of prime factors.
d Find the HCF of 350 and 112.
e Find the LCM of 350 and 112.

2 Copy and complete this multiplication table.

×	−6	−5	7
−10			
3	−18		
−7			

3 Are these calculations correct? If not, correct them.
a $(-5)^2 = -25$
b $-9 \times -11 = -99$
c $45 \div -9 = -6$
d $(-10)^3 = -1000$

4 Work out
a $40 \div -5$
b $-36 \div -6$
c $100 \div (2 - 7)$
d $(12 - -18) \div -3$

5 Solve these equations.
a $x^2 = 36$
b $x^2 + 16 = 0$
c $x^3 = 8$
d $x^3 + 27 = 0$

6 Work out
a $\sqrt{(-5)^2 - (-4)^2}$
b $\sqrt[3]{64} + \sqrt[3]{-64}$

7 Here is an expression: $x^3 + x^2$
Find the value of the expression when
a $x = 3$
b $x = -3$

8 Write as a single power of 8
a $8^2 \times 8^3$
b $8^8 \div 8^2$
c 1
d $(8^3)^3$

9 a Write 4^6 as a power of 2.
b Write 9^4 as a power of 3.