Contoh 1 (Dasar)

Hitunglah: $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

Pembahasan:

1. Substitusi: $\frac{\sqrt{4} - 2}{4 - 4} = \frac{0}{0}$ (Tak tentu).

2. Kali Sekawan: Kalikan dengan $\frac{\sqrt{x} + 2}{\sqrt{x} + 2}$.

   $\lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2}$

   $= \lim_{x \to 4} \frac{(\sqrt{x})^2 - 2^2}{(x - 4)(\sqrt{x} + 2)}$

   $= \lim_{x \to 4} \frac{x - 4}{(x - 4)(\sqrt{x} + 2)}$

3. Sederhanakan: Coret $(x - 4)$.

   $\lim_{x \to 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4}$

Jawaban: $\frac{1}{4}$

Contoh 2 (Akar di Pembilang)

Hitunglah: $\lim_{x \to 3} \frac{\sqrt{x + 1} - 2}{x - 3}$

Pembahasan:

1. Substitusi: $\frac{\sqrt{4} - 2}{3 - 3} = \frac{0}{0}$.

2. Kali Sekawan: Sekawan dari $(\sqrt{x + 1} - 2)$ adalah $(\sqrt{x + 1} + 2)$.

   $\lim_{x \to 3} \frac{(x + 1) - 4}{(x - 3)(\sqrt{x + 1} + 2)}$

   $= \lim_{x \to 3} \frac{x - 3}{(x - 3)(\sqrt{x + 1} + 2)}$

3. Hasil: $\frac{1}{\sqrt{3 + 1} + 2} = \frac{1}{4}$

LIVEWORKSHEETS

Jawaban: $\frac{1}{4}$

Contoh 3 (Akar di Penyebut)

Hitunglah: $\lim_{x \to 0} \frac{x}{\sqrt{1+x}-1}$

Pembahasan:

1. Kali Sekawan Penyebut: Kalikan dengan $(\sqrt{1+x}+1)$.
   $\lim_{x \to 0} \frac{x(\sqrt{1+x}+1)}{(1+x)-1}$
   $= \lim_{x \to 0} \frac{x(\sqrt{1+x}+1)}{x}$
2. Hasil: $\sqrt{1+0}+1 = 1+1 = 2$

Jawaban: 2

Contoh 4 (Akar dengan Koefisien)

Hitunglah: $\lim_{x \to 2} \frac{\sqrt{2x+5}-3}{x-2}$

Pembahasan:

1. Kali Sekawan: $\frac{\sqrt{2x+5}+3}{\sqrt{2x+5}+3}$.
   $\lim_{x \to 2} \frac{(2x+5)-9}{(x-2)(\sqrt{2x+5}+3)}$
   $= \lim_{x \to 2} \frac{2x-4}{(x-2)(\sqrt{2x+5}+3)}$
2. Faktorkan: $2x-4 = 2(x-2)$.
   $\lim_{x \to 2} \frac{2(x-2)}{(x-2)(\sqrt{2x+5}+3)} = \frac{2}{\sqrt{9}+3} = \frac{2}{6} = \frac{1}{3}$

Jawaban: $\frac{1}{3}$

Contoh 5 (Dua Bentuk Akar)

Hitunglah: $\lim_{x \to 1} \frac{\sqrt{2x-1}-\sqrt{x}}{x-1}$

LIVEWORKSHEETS

Pembahasan:

1. Kali Sekawan: $(\sqrt{2x-1}+\sqrt{x})$.
   $\lim_{x\to 1}\frac{(2x-1)-(x)}{(x-1)(\sqrt{2x-1}+\sqrt{x})}$
   $=\lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{2x-1}+\sqrt{x})}$
2. Hasil: $\frac{1}{\sqrt{2(1)-1}+\sqrt{1}}=\frac{1}{1+1}=\frac{1}{2}$

Jawaban: $\frac{1}{2}$

Latihan Soal: Limit Fungsi Aljabar (Perkalian Sekawan)

Selesaikan soal-soal berikut secara bertahap. Pastikan untuk selalu mencoba metode substitusi terlebih dahulu!

Level 1: Mudah (Dasar Perkalian Sekawan)

1. $\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3}$

2. $\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$

3. $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$

4. $\lim_{x\to 2}\frac{x-2}{\sqrt{x+7}-3}$

LIVEWORKSHEETS

5. $\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x-5}$

6. $\lim_{x \to 0} \frac{3x}{\sqrt{9+x}-3}$

7. $\lim_{x \to 1} \frac{1-x}{1-\sqrt{x}}$

Level 2: Sedang (Melibatkan Pemfaktoran/Koefisien)

8. $\lim_{x \to 4} \frac{\sqrt{2x+1}-3}{x-4}$

9. $\lim_{x \to 1} \frac{\sqrt{3x+1}-2}{x^2-1}$

10. $\lim_{x \to 2} \frac{x^2-4}{\sqrt{x+2}-2}$

LIVEWORKSHEETS

11. $\lim_{x \to -1} \frac{x+1}{\sqrt{x+5}-2}$

12. $\lim_{x \to 3} \frac{2-\sqrt{x+1}}{x-3}$

13. $\lim_{x \to 0} \frac{\sqrt{1+2x}-\sqrt{1-2x}}{x}$

14. $\lim_{x \to 7} \frac{x-7}{\sqrt{x+2}-3}$

Level 3: Sulit (Bentuk Kompleks)

LIVEWORKSHEETS

15. $\lim_{x \to 4} \frac{3 - \sqrt{x^2 - 7}}{x^2 - 16}$

16. $\lim_{x \to 2} \frac{\sqrt{x + 2} - \sqrt{3x - 2}}{x - 2}$

17. $\lim_{x \to 1} \frac{\sqrt{x^2 + 3} - 2}{x - 1}$

18. $\lim_{x \to 0} \frac{x^2}{\sqrt{x^2 + 1} - 1}$

19. $\lim_{x \to 3} \frac{\sqrt{2x - 2} - 2}{\sqrt{3x} - 3}$

LIVEWORKSHEETS

20. $\lim_{x \to 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}}$

LIVEWORKSHEETS