FINAL EXAM REVISION SHEET

Night Before the Exam

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INTERNATIONAL WORKSHEETS SERIES

This revision sheet contains the most important concepts, formulas, examples, and exam-style questions needed for the exam.

Good Luck!

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Lesson Objectives & Overview

This comprehensive revision covers essential Grade 10 Algebra 2 topics from lessons 3.2 to 3.5:

- **3.2 Functions:** Determine whether a relation is a function. Find the domain and range. Graph linear equations.
- **3.3 Rate of Change & Behavior:** Find the average rate of change from tables, graphs, and formulas. Determine where a function is increasing, decreasing, or constant. Locate local and absolute extrema.
- **3.4 Composition of Functions:** Combine functions using algebraic operations ($+, -, \times, \div$). Create, evaluate, and solve word problems using composite functions ($f \circ g$).
- **3.5 Transformation of Functions:** Describe and apply translations (vertical/horizontal), reflections (x-axis/y-axis), and dilations.

Key Vocabulary & Summary Notes

- **Function:** A relation where every input (x) has exactly one output (y). Fails the Vertical Line Test if it's not a function.
- **Domain & Range:** Domain is all possible x-values (inputs). Range is all possible y-values (outputs).
- **Extrema:** The highest (Maximum) or lowest (Minimum) points on a graph. Can be Local (relative to surrounding points) or Absolute (over the entire domain).
- **Composite Function:** Applying one function to the results of another, written as $(f \circ g)(x) = f(g(x))$.
- **Transformations:** Operations that alter the graph of a parent function (shift, stretch, reflect).

Essential Formula

1. Average Rate of Change Formula:

$$\text{Rate of Change} = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

2. Operations with Functions:

- Addition: $(f+g)(x) = f(x) + g(x)$
- Subtraction: $(f-g)(x) = f(x) - g(x)$
- Multiplication: $(fg)(x) = f(x) \cdot g(x)$
- Division: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$

3. Transformations of $f(x)$:

- **Vertical Shift:** $f(x) \pm k$ (Up $+$, Down $-$)
- **Horizontal Shift:** $f(x \pm h)$ (Left $+$, Right $-$)
- **Reflections:** $-f(x)$ (over x-axis), $f(-x)$ (over y-axis)

Quick Revision Tip ⋆

Inside vs. Outside: In transformations, anything grouped *inside* the parentheses with the x affects the graph horizontally and usually in the *opposite* way you expect (e.g., $x - 3$ moves RIGHT). Anything *outside* affects the graph vertically exactly as expected.

Worked Examples

Example 1: Function Operations and Composition Given $f(x) = 2x^2 + 1$ and $g(x) = 3x - 5$. Find $(f - g)(x)$ and $(f \circ g)(2)$.

- **Subtraction:** $(f - g)(x) = (2x^2 + 1) - (3x - 5) = 2x^2 - 3x + 6$.
- **Composition:** First find $g(2) = 3(2) - 5 = 1$. Then find $f(g(2)) = f(1) = 2(1)^2 + 1 = 3$.

Example 2: Average Rate of Change Find the average rate of change of $f(x) = x^2 - 4x$ on the interval $[-1, 3]$.

- Step 1: Find $f(-1) = (-1)^2 - 4(-1) = 1 + 4 = 5$.
- Step 2: Find $f(3) = (3)^2 - 4(3) = 9 - 12 = -3$.
- Step 3: Apply formula: $\frac{-3-5}{3-(-1)} = \frac{-8}{4} = -2$.

Example 3: Transformations Describe the transformations applied to the parent function $f(x) = \sqrt{x}$ to get $g(x) = -\sqrt{x+4} - 2$.

- $x + 4$ inside: Horizontal shift **Left 4 units**.
- Negative sign outside: **Reflection across the x-axis**.
- -2 outside: Vertical shift **Down 2 units**.

Common Mistake △

- **Composition Order:** $(f \circ g)(x)$ is NOT the same as $(f \cdot g)(x)$. The first is substituting $g(x)$ into $f(x)$, the second is multiplication. Also, $(f \circ g)(x) \neq (g \circ f)(x)$ in most cases!
- **Domain Restrictions:** When dealing with rational functions, remember you cannot divide by zero. When dealing with radicals, you cannot take the square root of a negative number.
- **Rate of Change:** Don't mix up the x and y values in the formula. It is always $\frac{\text{Change in } y}{\text{Change in } x}$.

Exam-Style Practice Questions

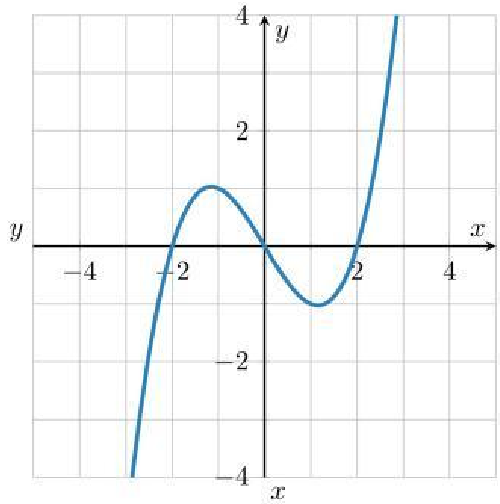
Q1. Domain, Range, and Functions (Multiple Choice):
Which of the following relations is NOT a function?

(A) $\{(2,3),(3,4),(4,5)\}$
(B) $\{(1,5),(2,5),(3,5)\}$
(C) $\{(4,1),(4,2),(5,3)\}$
(D) $\{(0,0),(1,1),(-1,1)\}$

Q2. Domain Analysis:
Determine the domain of the function $f(x) = \frac{3x}{x^2-9}$. Express your answer in interval notation.

Q3. Evaluating Composition:
Given $f(x) = 4x - 3$ and $g(x) = x^2 + 2$, evaluate $(g \circ f)(1)$.

Q4. Graphical Analysis:
Use the graph of the function $h(x)$ below to answer parts (a) and (b).



Note: The function shown is a cubic polynomial with turning points at approximately $x = -1.15$ *and* $x = 1.15$*.*
(a) Determine the approximate intervals where the function is increasing and decreasing.
(b) Identify the approximate coordinates of any local maxima and local minima.

Q5. Average Rate of Change from a Table:
The table below shows the distance $d(t)$ in meters a car travels in t seconds.

Time (t)	0	2	4	6	8
Distance ($d(t)$)	0	10	30	60	100

Find the average rate of change in distance between $t = 2$ and $t = 6$ seconds. Include units in your answer.

Q6. Algebraic Operations on Functions:
Let $p(x) = x^2 - 1$ and $q(x) = x + 1$. Find and simplify the expression for $\left(\frac{p}{q}\right)(x)$ and state its domain.

Q7. Writing Composite Functions:
Given $f(x) = \sqrt{x+2}$ and $g(x) = 3x - 5$. Find an algebraic expression for $(f \circ g)(x)$.

Q8. Decomposing Functions (Higher Order Thinking):
Find two functions $f(x)$ and $g(x)$ such that $h(x) = (4x + 7)^5$ can be expressed as $h(x) = f(g(x))$.

Q9. Transformation Descriptions:
Describe the sequence of transformations required to graph $g(x) = -2|x-3|+1$ starting from the parent function $f(x) = |x|$.

Q10. Writing Transformed Equations:
The graph of the parent function $f(x) = x^2$ is translated 5 units left, reflected over the x-axis, and translated 3 units down. Write the equation of the new function $g(x)$.

Q11. Graphing Linear Equations:
Graph the linear equation $3x - 2y = 6$ on a coordinate plane. (State the x-intercept and y-intercept).

Q12. Average Rate of Change from an Expression:
Find the average rate of change of the function $f(x) = 2x^2 + 3x$ on the interval $[a, a+h]$. (Simplify your answer completely).

Q13. Word Problem: Rate of Change:
The population of a town was 12,000 in the year 2010. By 2020, the population had grown to 18,500. Assuming the growth is relatively linear, what is the average rate of change of the population per year?

Q14. Word Problem: Composition (Success Criteria Green):
A clothing store is having a 20% off sale on all items. You also have a loyalty coupon for $\$10$ off any purchase. Let x be the original price of an item.

(a) Write a function $d(x)$ that represents the price after the 20% discount.
(b) Write a function $c(x)$ that represents the price after applying the $\$10$ coupon.
(c) If the store applies the discount first, then the coupon, write the composite function. Find the final price of a $\$60$ jacket.

Q15. Absolute Extrema vs Local Extrema:

Explain the difference between a local minimum and an absolute minimum using a sketch or a short written explanation.

Answer Key

A1. **(C)** $\{(4,1),(4,2),(5,3)\}$. It is not a function because the input 4 has two different outputs (1 and 2), failing the definition of a function.

A2. The denominator cannot be zero. $x^2-9\neq 0 \implies x^2\neq 9 \implies x\neq 3, x\neq -3$.
Interval Notation: $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$.

A3. First, evaluate $f(1)=4(1)-3=1$.
Next, plug this result into $g(x)$: $g(1)=(1)^2+2=3$.
Final Answer: 3.

A4. **(a)** Increasing: $(-\infty,-1.15)\cup(1.15,\infty)$. Decreasing: $(-1.15,1.15)$.
(b) Local Max at approximately $x\approx -1.15$ (positive y-value). Local Min at approximately $x\approx 1.15$ (negative y-value).

A5. Apply Rate of Change formula: $\frac{d(6)-d(2)}{6-2}=\frac{60-10}{4}=\frac{50}{4}=12.5$.
Final Answer: 12.5 meters per second.

A6. $\left(\frac{p}{q}\right)(x)=\frac{x^2-1}{x+1}=\frac{(x-1)(x+1)}{x+1}=x-1$.
Domain: All real numbers except $x\neq -1$ (since the original denominator $q(x)$ cannot be zero).

A7. $(f\circ g)(x)=f(g(x))=f(3x-5)=\sqrt{(3x-5)+2}=\sqrt{3x-3}$.

A8. **Inner function** $g(x)=4x+7$.
Outer function $f(x)=x^5$.
Check: $f(g(x))=f(4x+7)=(4x+7)^5$.

A9.
1. Horizontal shift **Right 3 units**.
2. **Vertical stretch** by a factor of 2.
3. **Reflection** across the x-axis.
4. Vertical shift **Up 1 unit**.

A10. $g(x)=-(x+5)^2-3$.

A11. Find intercepts:
x-intercept (set $y=0$): $3x=6 \implies x=2 \implies (2,0)$.
y-intercept (set $x=0$): $-2y=6 \implies y=-3 \implies (0,-3)$.
Plot these two points and draw a line through them.

A12. Formula: $\frac{f(a+h)-f(a)}{h}$.
$f(a+h)=2(a+h)^2+3(a+h)=2(a^2+2ah+h^2)+3a+3h=2a^2+4ah+2h^2+3a+3h$.
$f(a)=2a^2+3a$.
Numerator: $(2a^2+4ah+2h^2+3a+3h)-(2a^2+3a)=4ah+2h^2+3h$.
Divide by h: $\frac{h(4a+2h+3)}{h}=4a+2h+3$.

A13. $\frac{\text{Change in Population}}{\text{Change in Time}}=\frac{18500-12000}{2020-2010}=\frac{6500}{10}=650$ people/year.

A14. **(a)** $d(x) = x - 0.20x = 0.80x$
(b) $c(x) = x - 10$
(c) Apply discount first, then coupon means $c(d(x))$.
$c(d(x)) = c(0.80x) = 0.80x - 10$.
For a $\$60$ jacket: $0.80(60) - 10 = 48 - 10 = 38$. **Final price is $38**.

A15. A **local minimum** is the lowest point in a specific, small section of a graph (a "valley"). An **absolute minimum** is the lowest point on the entire graph across its whole domain. A function can have multiple local minima, but only one absolute minimum value.