

Activity: Scalar Triple Products and Volume

Question 1: Guided Example Find the volume of the parallelepiped with edges $\mathbf{a} = \langle 2, 1, 0 \rangle$, $\mathbf{b} = \langle 0, 3, 1 \rangle$, and $\mathbf{c} = \langle 1, 0, 4 \rangle$.

Step 1: Set up the determinant

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 4 \end{vmatrix}$$

Step 2: Expand along the first row.

$$2 \begin{vmatrix} 3 & 1 \\ 0 & 4 \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ 1 & 4 \end{vmatrix} + 0 \begin{vmatrix} 0 & 3 \\ 1 & 0 \end{vmatrix}$$

Step 3: Calculate the 2x2 minors $2(12 - 0) - 1(0 - 1) + 0 = 2(\text{_____}) - 1(\text{_____})$

Step 4: Result Scalar Triple Product = _____ Final Volume = _____ cubic units