رياضيات 1

Mathematics 1

The Functions

اعداد

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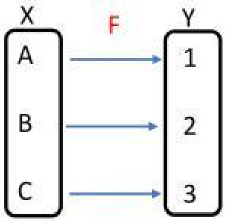
* Defintion :

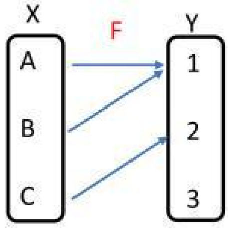
A functions is a relation between the elements of two unempty sets ,where every element in a set has only one image in the other set.

this can be written in the form $y = \mathrm{f}(x)$ *or* $\mathrm{f}:\mathrm{X} \to \mathrm{Y}$

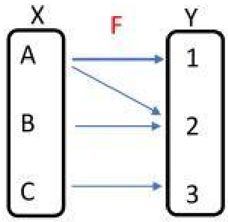
x is independent variable, y is dependen variable.



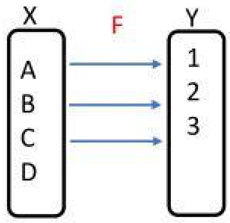
Function



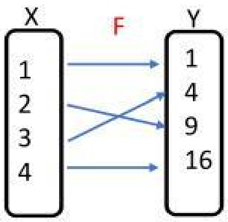
Function



Not Function



Not Function



$\mathrm{f}:\mathrm{X} \to \mathrm{Y}$, $f(x) = y$

$1 \to 1$ $f(1) = 1$

$2 \to 9$ $f(2) = 9$

$3 \to 4$ $f(3) = 4$

$4 \to 16$ $f(4) = 16$

*Domain of the function :

where X is the domain of the function f and may be denoted by D_f .

Y is the Co-domain of f.

*Range of the function :

the range is a set of all image of the elements of the domain and it is denoted by R_f .

we note that $R_f \subset Y$

*Remarks to Find the Nutral Domain :

1.For rational functions the domain is R except any number make the dominator to be zero i.e. domain =R- {zeros of dominator}.

2.When the function contain square root or any even root $\sqrt[4]{\ }$, $\sqrt[6]{\ }$ ....*hence the domain will be R* except the numbers which make the value under the root sign to be negative value.

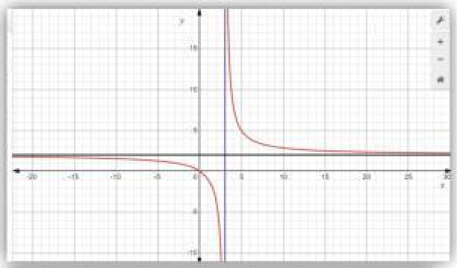
Example : Find the domain and range of the functions

1) $f(x)=\dfrac{2x}{x-3}$ R- {zeros of dominator}

$x-3=0 \rightarrow x=3 \rightarrow D_f = R-\{3\}$

$y=\dfrac{2x}{x-3} \rightarrow yx-3y=2x \rightarrow yx-2x=3y \rightarrow x(y-2)=3y \rightarrow x=\dfrac{2y}{y-2}$

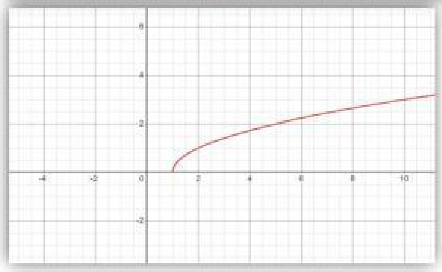
$R_f = R-\{2\}$



2) $f(x)=\sqrt{x-1}$ the value under the root ≥ 0

$x-1\geq 0 \rightarrow x\geq 1 \rightarrow D_f=[1,\infty[\quad or \quad [1,\infty)$

$R_f=[0,\infty[$



3) $y=\dfrac{1}{\sqrt{x^2-1}}$ the value under the root >0

$x^2-1>0 \rightarrow x^2>1 \rightarrow \pm x>1$

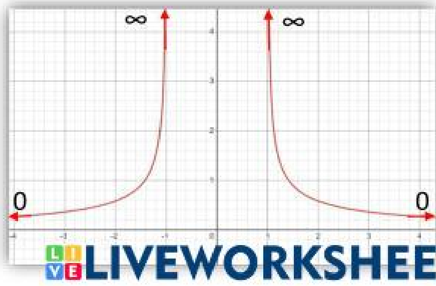
$x>1 \quad or \quad -x>1$

$x>1 \quad or \quad x<-1$

$D_f = R-[-1,1]$

$at\ x=\pm 1 \rightarrow y=\infty \quad , \quad x=\pm\infty \rightarrow y=0$

$R_f=]0,\infty[$



4) $f(x)=\sqrt{4-\sqrt{x}}$

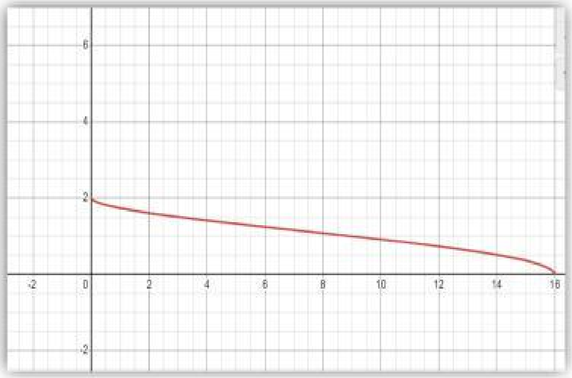
$\sqrt{x} \rightarrow \boxed{x \geq 0}$

$\sqrt{4-\sqrt{x}} \rightarrow 4-\sqrt{x} \geq 0 \rightarrow 4 \geq \sqrt{x} \rightarrow \boxed{16 \geq x}$

$0 \leq x \leq 16$, $D_f = [0,16]$

$at\ x=0 \rightarrow y=2$, $x=16 \rightarrow y=0$

$R_f = [0,2]$



5) $y=\sqrt{4-x^2}$

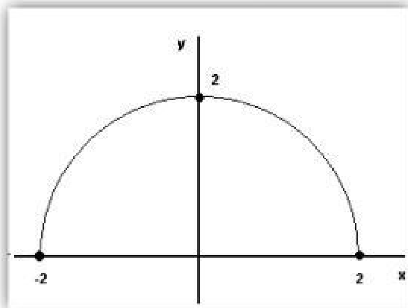
$4-x^2 \geq 0 \rightarrow 4 \geq x^2 \rightarrow 2 \geq \pm x$

$2 \geq x$ or $2 \geq -x$

$-2 \leq x$

$-2 \leq x \leq 2 \Rightarrow D_f = [-2,2]$

$R_f = [0,2]$



*) $y=\sqrt{4-x^2} \Rightarrow y$ is always positive

$y^2 = 4-x^2 \rightarrow x^2+y^2=4$

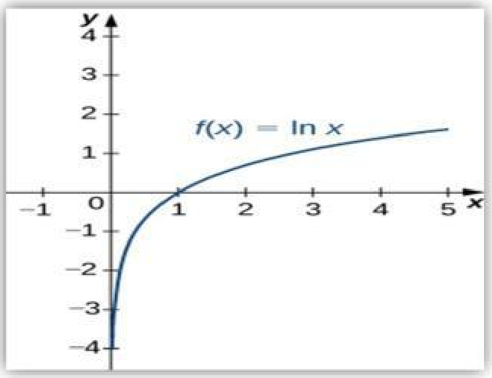
$\boxed{x^2+y^2=a^2}$

*) $y = \ln x$

the value inside $\ln > 0$

$x > 0 \rightarrow D_f =]0, \infty[$

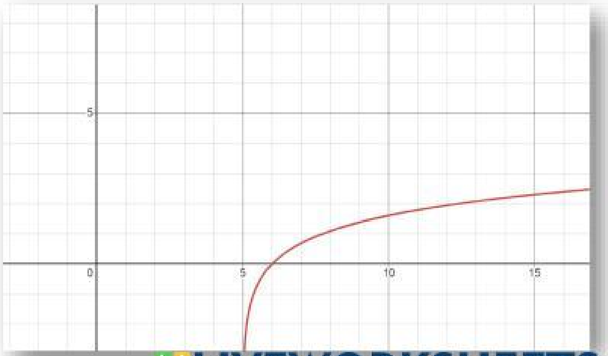
$R_f =]-\infty, \infty[= R$



4) $y = \ln(x-5)$

$x - 5 > 0 \rightarrow x > 5 \rightarrow D_f =]5, \infty[$

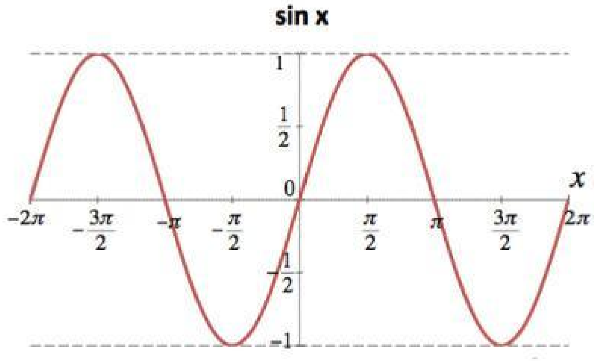
$R_f =]-\infty, \infty[= R$



Example: Find the range of the functions

1) $y = \dfrac{1}{2 - \sin x}$

$2y - y\sin x = 1 \rightarrow y\sin x = 2y - 1 \rightarrow \sin x = \dfrac{2y-1}{y}$



$\sin x = 2 - \dfrac{1}{y}$

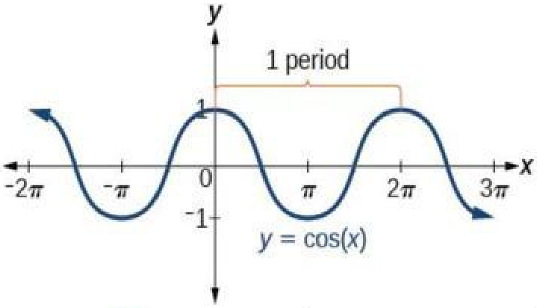
$-1 \le \sin x \le 1$

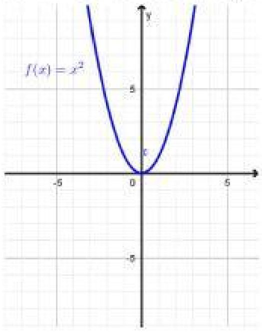
$-1 \le 2 - \dfrac{1}{y} \le 1 \rightarrow -3 \le \dfrac{-1}{y} \le -1 \rightarrow 3 \ge \dfrac{1}{y} \ge 1$

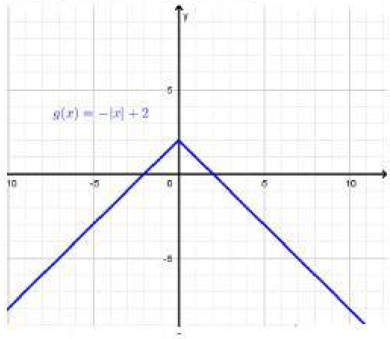
$\dfrac{1}{3} \le y \le 1 \quad \Rightarrow R_f = [\dfrac{1}{3}, 1]$

Even and Odd Functions

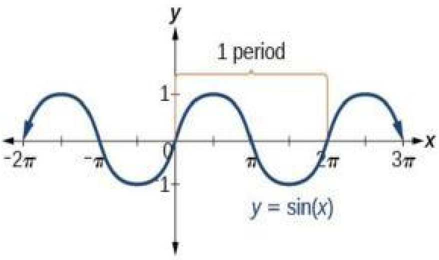
$f(x)$ is an even function if $\boxed{f(-x) = f(x)}$ and has Y-axis as a symmetric line of it

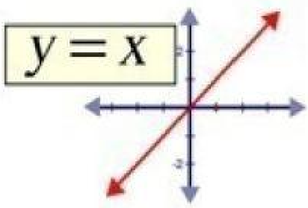


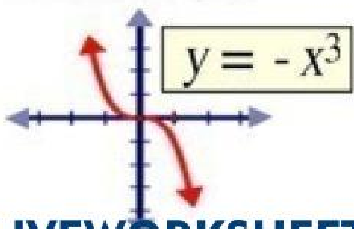




$f(x)$ is an odd function if $\boxed{f(-x) = -f(x)}$ and the curve of $f(x)$ *is* symmetric about the origin







Ex : Check the following functions even, odd, or not : -

1) $f(x) = x^2 - 1$

$$f(-x) = (-x)^2 - 1 = x^2 - 1 = f(x)$$

$\therefore$ *f(x) is even function*

2) $f(x) = x^2 + x$

$$f(-x) = (-x)^2 + (-x) = x^2 - x \neq f(x)$$

and $f(-x) \neq -f(x)$

$\therefore$ *f(x) neither odd nor even function*

3) $f(x) = \dfrac{x^3 + 2x}{x^4 - 3x^2 + 6}$

$$f(-x) = \frac{(-x)^3 + 2(-x)}{(-x)^4 - 3(-x)^2 + 6} = \frac{-(x^3 + 2x)}{x^4 - 3x^2 + 6} = -f(x)$$

$\rightarrow f(-x) = -f(x)$

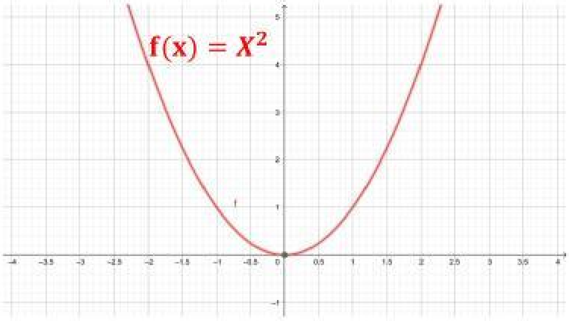
$\therefore$ *f(x) is odd function*

Types of the function

1. One to one function (injective)

$\forall x \in D_f \ , \ x_1 \neq x_2 \ \Rightarrow f(x_1) \neq f(x_2)$ $\Rightarrow \therefore f(x_1) = f(x_2) \Rightarrow \ x_1 = x_2$

$Ex: \ f(x) = x^2$



$let \ \ f(x_1) = f(x_2)$

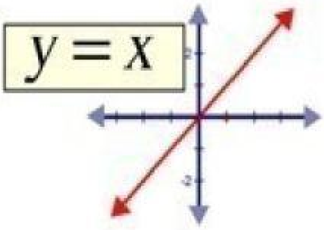
$x^2{}_1 = x^2{}_2 \ \Rightarrow \ x^2{}_1 - x^2{}_2 = 0$

$(x_1 - x_2)(x_1 + x_2) = 0$

$x_1 = x_2 \quad or \qquad x_1 = -x_2$

$f(x)$ is not one to one function

$Ex: \ f(x) = x$



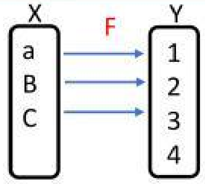
$let \ \ f(x_1) = f(x_2)$

$x_1 = x_2$

$f(x)$ is one to one function

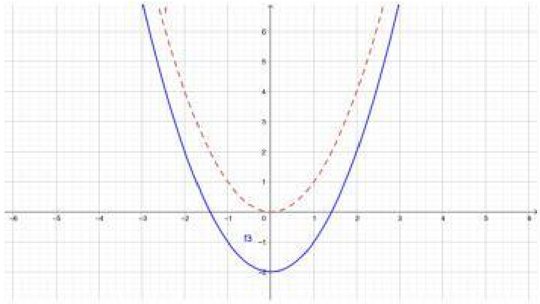
2. Onto function (surjective)

Range = Co_domain ($R_f = Y$)



(Co_domain) $Y = \{1,2,3,4\}$, (Range) $R_f = \{1,2,3\} \Rightarrow \because Y \neq R_f$

$\therefore f(x)$ is not Onto function



Ex: Is $f(x) = x^2 - 2$ Onto where $f: R \to R$?

($f: domain \to Co\_domain$)

$R_f = [-2, \infty[$, $Y = R$ $\to R_f \neq Y$ $\to \therefore f(x)$ is not onto function

Ex: Is $f(x) = x^2 - 2$ Onto where $f: R \to [-2, \infty[$?

$R_f = [-2, \infty[$, $Y = [-2, \infty[$ $\to R_f = Y$ $\to \therefore f(x)$ is onto function

3. Bijective function

if it is one to one (injective) and onto (surjective)

*Invers of a Function:

A function $f(x)$ has an invers function $f^{-1}(x)$ if and only if it is bijective (or at least is one to one)

Note that :

1. $f^{-1}(x) \neq \frac{1}{f(x)}$

2. The domain of $f^{-1}(x)$ is the co_domain of $f(x)$ and its range is the domain of f

3. The function and its invers are symmetric at $y = x$

Ex : Show that $f(x) = 2x + 1$ has invers and find it

$D_f = R$, $R_f = R$

let $f(x_1) = f(x_2)$

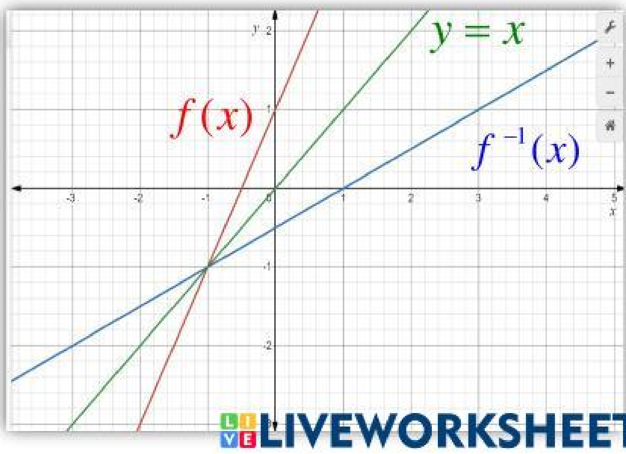
$2x_1 + 1 = 2x_2 + 1 \Rightarrow 2x_1 - 2x_2 = 0 \Rightarrow 2(x_1 - x_2) = 0$

$\therefore x_1 = x_2 \Rightarrow f(x)$ is one to one function $\Rightarrow f(x)$ has invers

$\boxed{f(x) = y = 2x + 1} \Rightarrow x = \frac{y-1}{2}$

$f^{-1}(y) = \frac{y-1}{2}$

$\boxed{f^{-1}(x) = \frac{x-1}{2}}$



Let f , g two functions

$f(g(x)) = x$ and $g(f(x)) = x$

$\therefore$ g is the invers of f and f is the invers of g

$f(x) = 2x + 1$, $f^{-1}(x) = \frac{x-1}{2} = g(x)$

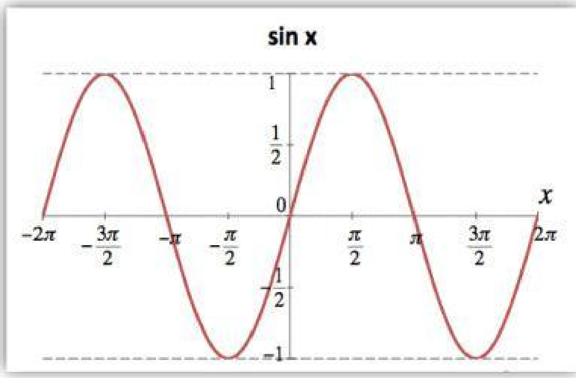
$f(g(x)) = 2(\frac{x-1}{2}) + 1 = x$ and $g(f(x)) = \frac{(2x+1)-1}{2} = x$

$\therefore$ g is the invers of f and f is the invers of g

Properties of Function

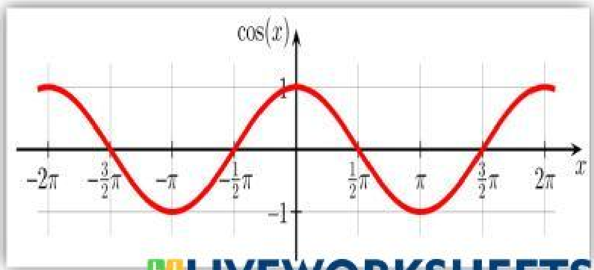
* **Periodic Function**

$$f(x \pm p) = f(x)$$



$Ex: \ f(x) = \sin(x)$

$$p = 2\pi \quad \rightarrow \quad \sin(x \pm 2\pi) = \sin(x)$$



$Ex: \ f(x) = \cos(x)$

$$p = 2\pi \quad \rightarrow \quad \cos(x \pm 2\pi) = \cos(x)$$

Thank you