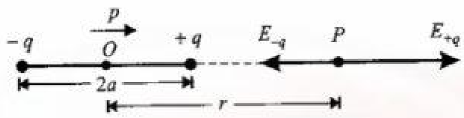
Unit-1 Electrostatics

(Electric Charges and Fields)

1. Derive the expression for electric field intensity at a point the axial line of a dipole.

As shown in the figure, consider an electric dipole of charges $-q$ and $+q$, separated by distance ____and placed in vacuum. Let P be point on the axial line at a distance r from the centre O of the dipole on the side of the charge $+q$.



Electric field due to charge $-q$ at a point P is

$$\vec{E}_{-q} = \frac{-q}{4\pi\varepsilon_0(\underline{\qquad})^2}\hat{p}$$

Where $\hat{p}$ is a unit vector along the dipole axis from $-q$ to $+q$.

Electric field due to charge $+q$ at a point P is

$$\vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0(\underline{\qquad})^2}\hat{p}$$

Hence the resultant electric field at point P is

$$\vec{E}_{axial} = \vec{E}_{-q} + \vec{E}_{+q}$$

$$= \frac{q}{4\pi\varepsilon_o}\left[\frac{1}{(\underline{\qquad})^2} - \frac{1}{(\underline{\qquad})^2}\right]\hat{p} = \frac{q}{4\pi\varepsilon_o}\cdot\frac{\underline{\qquad}}{(\underline{\quad}^2 - \underline{\quad}^2)^2}\hat{p}$$

$$\boxed{\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_o}\cdot\frac{\underline{\qquad}}{(\underline{\quad}^2 - \underline{\quad}^2)^2}\hat{p}}$$

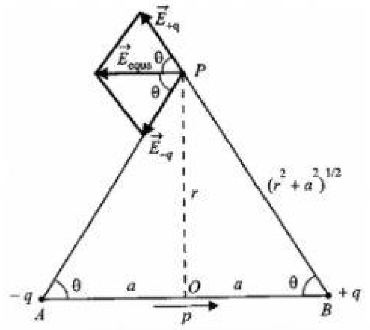
Here $p = qx2a$=dipole moment. For $r \gg a, a^2$ can be neglected compared to r^2.

$$\therefore \boxed{\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_o}\cdot\frac{\underline{\quad}}{\underline{\quad}^3}\hat{p}}$$

Electric field at any point on the axis of the dipole acts along the dipole axis from ________ to _________ charge, i.e. in the direction of ______________.

2. **Derive the expression for the electric field intensity at any point along the equatorial line of an electric dipole.**

As shown in the figure, consider an electric dipole of charges $-q$ and $+q$, separated by distance _____ and placed in vacuum. Let P be point on the equatorial line of the dipole at distance r from it. i.e. $OP = r$



Electric filed at point P due to $+q$ charge is

$$\vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0(\_\_^2 + \_\_^2)} \text{ directed along } BP.$$

Electric filed at point P due to $-q$ charge is

$$\vec{E}_{-q} = \frac{q}{4\pi\varepsilon_0(\_\_^2 + \_\_^2)} \text{ directed along } P.$$

Clearly, the magnitudes of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ are equal. So, the components of $\vec{E}_{+q}$ and $\vec{E}_{-q}$ normal to the dipole axis will cancel out. The components parallel to the dipole axis will add up. The total electric field $\vec{E}_{eq}$ is opposite to $\vec{p}$.

$$\therefore \vec{E}_{eq} = -\left(E_{-q}\,\_\_\_\,\theta + E_{+q}\,\_\_\_\,\theta\right)\hat{p}$$

$$\vec{E}_{eq} = -\_\_\,\vec{E}_{+q}\,\_\_\_\,\theta\ \hat{p} \qquad (\because\ E_{+q} = E_{-q})$$

$$or, \vec{E}_{eq} = -2\,\frac{q}{4\pi\varepsilon_0(\_^2 + \_^2)}\,\_\_\,\theta\ \hat{p}$$

Now, from fig. ___ $\theta = \frac{a}{\sqrt{r^2+a^2}}$,

$$\therefore \vec{E}_{eq} = -2\,\frac{q}{4\pi\varepsilon_0(\_\_^2 + \_\_^2)}\frac{a}{\sqrt{r^2+a^2}}\hat{p}$$

$$or, \boxed{\vec{E}_{eq} = -\frac{1}{4\pi\varepsilon_o}\cdot\frac{\_\_\_}{(\_\_^2 + \_\_^2)^{\frac{3}{2}}}\hat{p}} \quad where\ \_\_\_ = 2qa, is\ electric\ dipole\ moment$$

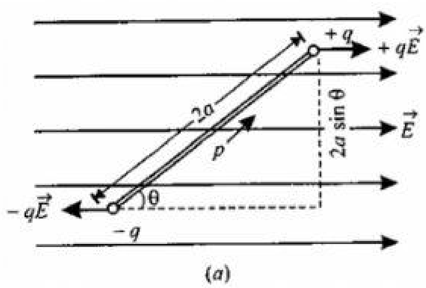
If the point P is located far away from the dipole, $r \gg a$, then

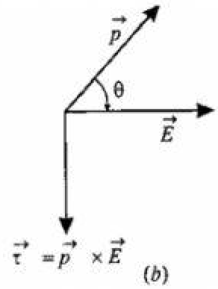
$$\boxed{\vec{E}_{eq} = -\frac{1}{4\pi\varepsilon_o}\cdot\frac{\_\_}{\_\_^3}\hat{p}}$$

The direction of electric field at any point on the equatorial line of the dipole will be antiparallel to the dipole moment $\hat{p}$.

The electric field intensity due to a short dipole at a distance 'r' along its axis is ______ the intensity at the same distance along the equatorial axis.

3. **Derive the expression for the torque acting on an electric dipole, when held in a uniform electric field. (net translational force is zero)**





Consider an electric dipole consisting of charges $+q$ and $-q$ and length ___ placed in a uniform electric field $\vec{E}$, making an angle θ with it. It has a dipole moment of magnitude,

$$p = \_\_\_\_$$

Force exerted on charge $+q$ by field $\vec{E} = +\_\vec{E}$ along $\vec{E}$

Force exerted on charge $-q$ by field $\vec{E} = -\_\_\vec{E}$ opposite to $\vec{E}$

$$\therefore \vec{F}_{total} = +\_\_\vec{E} - \_\_\vec{E} = \_\_\_\_$$

Hence net translating force on dipole in a uniform electric field is _____. But the two equal and opposite force act at different points on the dipole. So, they form a couple and exert _______.

_______ = either force x perpendicular distance between the two forces

$$i.e. \tau = \_E \times 2a\_\_\_\_\_\theta = (\_\_ x\ 2a)E\_\_\_\_\theta$$

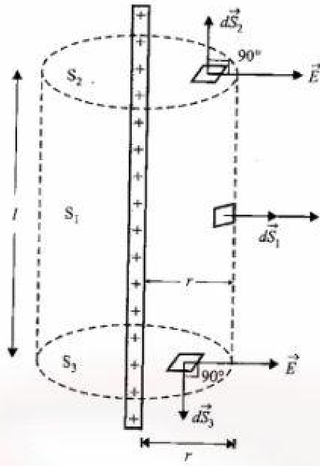
$$\boxed{\tau = \_\_\_E\_\_\_\_\theta}$$

As the direction of torque $\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$, we can write

$$\boxed{\vec{\tau} = \overrightarrow{\_\_} \times \overrightarrow{\_\_}}$$

As shown in the figure the direction of torque is that in which a right-handed screw would advance when rotated from $\overrightarrow{\_\_}$ to $\overrightarrow{\_\_}$.

4. **Derive the expression for the electric field intensity due to infinitely long, straight wire of linear charge density $\lambda\ Cm^{-1}$.**



Consider a thin infinitely long straight wire having a uniform - _________ charge density $\lambda\ Cm^{-1}$. By symmetry, the field $\vec{E}$ of the line charge is directed radially _________and its magnitude is same at all points equidistant from the line charge. Here we choose a _________ Gaussian surface of radius *r*, length *l* and with its axis along the line charge.

As shown in the figure the Gaussian surface has curved surfaces S_1 and flat circular ends S_2 and S_3.

Since $\overrightarrow{dS_1} \parallel \vec{E}, \overrightarrow{dS_2} \perp \vec{E}\ and\ \overrightarrow{dS_3} \perp \vec{E}$ only the curved surface contributes towards the total flux.

$$\emptyset_E = \oint_S \vec{E}.\overrightarrow{ds} = \int_{S_1} \vec{E}.\overrightarrow{ds_1} + \int_{S_2} \vec{E}.\overrightarrow{ds_2} + \int_{S_3} \vec{E}.\overrightarrow{ds_3}$$

$$\emptyset_E = \int_{S_1} EdS_1 \_\_\_\_ 0^o + \int_{S_2} EdS_2 \_\_\_\_ 90^o + \int_{S_3} EdS_3 \_\_\_\_ 90^o$$

$$\emptyset_E = E\int_{S_1} dS_1 = E\ x\ area\ of\ curved\ surface$$

$$\emptyset_E = E\ x\ 2\pi\_\_$$

Charged enclosed by the Gaussian surface is $q = \lambda\_\_$

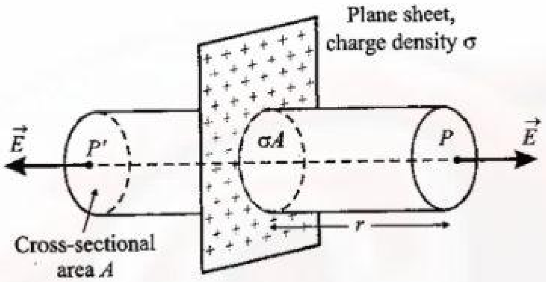
Using Gauss's theorem, $\emptyset_E = \frac{}{\varepsilon_o}$, we get

$$E.2\pi\_\_ = \frac{\lambda\_\_}{\varepsilon_o}$$

$$or, \boxed{E = \frac{\lambda}{2\pi\varepsilon_o\_\_\_}}$$

5. **Derive the expression for the electric field intensity at a point near a thin infinite plane sheet of charge density $\sigma\ Cm^{-2}$.**

As shown in the figure consider a thin, infinite plane sheet of charge with uniform _______ charge density σ. We have to calculate its electric field at a point P at distance r from it. By symmetry, electric field $\vec{E}$ points ________ normal to the sheet. Also, it must have same magnitude and opposite direction at two points P and $P`$ equidistant from the sheet and on opposite sides.



Here we consider a ___________ Gaussian surface of cross-sectional area A and length $2r$ with its axis perpendicular to the sheet. As the lines of force are parallel to the curved surface of the cylinder, the flux through the curved surface is ______. The flux through the plane end faces of the cylinder is

$$\emptyset_E = EA + EA = 2EA$$

Using Gauss's theorem, $\emptyset_E = \frac{\_\_}{\varepsilon_o}$, we get

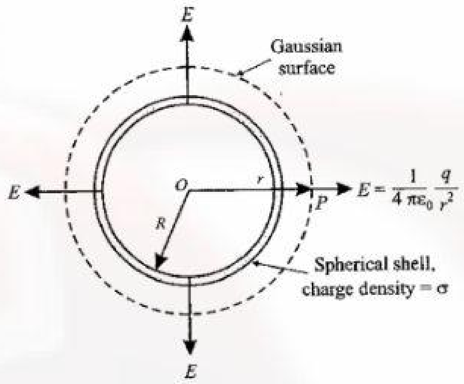
$$\_\_EA = \frac{\sigma\_\_}{\varepsilon_o}$$

$$or, \boxed{E = \frac{\sigma}{\_\_\_\varepsilon_o}}$$

6. Calculate the electric field due to a uniformly charged spherical shell at a point (i) outside the shell, (ii) on the shell and (iii) inside the shell.

Consider a thin spherical shell of charge of radius R with uniform ________ charge density σ. From symmetry, we see that the electric field $\vec{E}$ at any point is _______ and has same magnitude at points equidistant from the centre. To determine electric field at any point at a distance r from O, we choose a concentric sphere of radius r as the __________ surface.

(i) When point P lies outside the spherical shell: The total charge q inside the Gaussian surface is the charge on the shell of radius R and area $4\pi R^2$.



$$\therefore q = 4\pi R^2\sigma$$

Flux through the Gaussian surface, $\phi_E = E \times 4\pi r^2$

Using Gauss's theorem, $\phi_E = \frac{\quad}{\varepsilon_o}$, we get

$$E\ x\ 4\pi r^2 = \frac{\quad}{\varepsilon_o}$$

$$or, E = \frac{1}{4\pi\varepsilon_o}.\frac{\quad}{r^2}$$

For points outside the shell, the field due to uniformly charged shell is as if the entire charge of shell is concentrated at its ________.

(ii) When point P lies on the spherical shell: The Gaussian surface just encloses the charged spherical shell. Applying Gauss's theorem, for $r=R$

$$E\ x\ 4\pi R^2 = \frac{\quad}{\varepsilon_o}$$

$$or, E = \frac{\quad}{4\pi\varepsilon_o R^2}$$

$$or, \boxed{E = \frac{\sigma}{\varepsilon_o}} \qquad since\ q = 4\pi R^2\sigma$$

(iii) When point P lies inside the spherical shell: For $r<R$, the charge enclosed by Gaussian surface is _____

Flux through the Gaussian surface, $\phi_E = E\ x\ 4\pi r^2$

Using Gauss's theorem, $\phi_E = \frac{\quad}{\varepsilon_o}$, we get

$$E\ x\ 4\pi r^2 = \_\_\_\_\ or\ E = \_\_\_$$

Hence, electric field due to a uniformly charged spherical shell is _____at all points inside the shell.