

8. Consider the functions $f(x)$, $g(x)$, $h(x)$ as given below. Show that $(f \circ g) \circ h = f \circ (g \circ h)$ in each case.

(i) $f(x) = x - 1$, $g(x) = 3x + 1$ and $h(x) = x^2$

(ii) $f(x) = x^2$, $g(x) = 2x$ and $h(x) = x + 4$

(iii) $f(x) = x - 4$, $g(x) = x^2$ and $h(x) = 3x - 5$

8. கொடுக்கப்பட்ட $f(x)$, $g(x)$, $h(x)$ ஆகியவற்றைக் கொண்டு $(f \circ g) \circ h = f \circ (g \circ h)$ எனக் காட்டுக.

(i) $f(x) = x - 1$, $g(x) = 3x + 1$ மற்றும் $h(x) = x^2$

(ii) $f(x) = x^2$, $g(x) = 2x$ மற்றும் $h(x) = x + 4$

(iii) $f(x) = x - 4$, $g(x) = x^2$ மற்றும் $h(x) = 3x - 5$

Solution :

(i) Given $f(x) = \text{---}$,

$g(x) = \text{---}$, $h(x) = \text{---}$

To verify : $(f \circ g) \circ h = f \circ (g \circ h)$

$(f \circ g)(x) = f(\text{---})$

$= f(\text{---})$

$= \text{---} + 1$

$= 3x$

$\therefore ((f \circ g) \circ h)(x) = (f \circ g)(\text{---})$

$= (f \circ g)(\text{---})$

$= 3x^2 \quad \rightarrow (1)$

$(g \circ h)(x) = g(\text{---})$

$= g(\text{---})$

$= 3\text{---} + 1$

$\therefore (f \circ (g \circ h))(x) = f(\text{---})$

$= f(\text{---})$

$= \text{---} - 1$

$= 3x^2 \quad \rightarrow (2)$

$\therefore$ From (1) and (2),

$(f \circ g) \circ h = f \circ (g \circ h)$

(ii) Given $f(x) = x^2$, $g(x) = 2x$, $h(x) = x + 4$

$(f \circ g)(x) = f(\text{---})$

$= f(\text{---})$

$= (\text{---})^2$

$= 4x^2$

$\therefore ((f \circ g) \circ h)(x) = (f \circ g)(h(x))$

$= (f \circ g)(\text{---})$

$= 4(x + 4)^2 \quad \rightarrow (1)$

$(g \circ h)(x) = g(\text{---})$

$= g(\text{---})$

$= 2(\text{---})$

$= 2x + 8$

$\therefore (f \circ (g \circ h))(x) = f(\text{---})$

$= (2x + 8)^2$

$= (2(x + 4))^2$

$= 4(x + 4)^2 \quad \rightarrow (2)$

$\therefore$ From (1) and (2),

$(f \circ g) \circ h = f \circ (g \circ h)$.

(iii) Given $f(x) = x - 4$, $g(x) = x^2$, $h(x) = 3x - 5$

$(f \circ g)(x) = f(\text{---})$

$= f(\text{---})$

$= \text{---} - 4$

$\therefore ((f \circ g) \circ h)(x) = (f \circ g)(\text{---})$

$= (f \circ g)(\text{---})$

$= \text{---} - 4 \quad \rightarrow (1)$

$(g \circ h)(x) = g(\text{---})$

$= g(\text{---})$

$= (\text{---})^2$

$\therefore (f \circ (g \circ h))(x) = f(g \circ h)(x)$

$= f(3x - 5)^2$

$= \text{---} - 4 \quad \rightarrow (2)$

$\therefore$ From (1) and (2),

$(f \circ g) \circ h = f \circ (g \circ h)$.

