

MULTIVARIABLES FUNCTIONS (DOMAIN AND RANGE)

Example 2

$$f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$$

Determine the domain and the range of f .

1) Any restriction?

2) How many restriction?

Solution

➤ One restriction: $1 - x^2 - y^2 - z^2 \geq 0$

The domain is

step: $-x^2 - y^2 - z^2 \geq -1$

Domain: $D_f = \{ (x, y, z) \mid x, y, z \in \mathbb{R}, x^2 + y^2 + z^2 \leq 1 \}$

➤ The range is

Range: $R_f = \{ w \mid w \in \mathbb{R}, 0 \leq w \leq 1 \}$

Please show the steps during quizzes, tests or final exam

How?
Please show the steps during quizzes, tests or final exam

FILL IN THE BLANK

Explanation for Example 2

Range: $R_f = \{ w \mid w \in \mathbb{R}, 0 \leq w \leq 1 \}$

$$w = \sqrt{1 - x^2 - y^2 - z^2}$$

Rewrite add bracket: $w = \sqrt{1 - (x^2 + y^2 + z^2)}$

$$x^2 + y^2 + z^2 \geq \boxed{}$$

Fill in the blank

min value for $x^2 + y^2 + z^2$ is $\boxed{} \Rightarrow w = \sqrt{1 - ()} = \sqrt{1} = 1$

($\boxed{}$ because $x^2 + y^2 + z^2$ always positive)

max value for $x^2 + y^2 + z^2$ is $\boxed{} \Rightarrow w = \sqrt{1 - ()} = \sqrt{0} = 0$

($\boxed{}$ because $x^2 + y^2 + z^2 \leq \boxed{}$ for real number)

Thus w is between 0 and 1 $\Rightarrow 0 \leq w \leq 1$

How?
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