

Question 15

Find the equation of the lines which pass through the point of intersection of the pair of lines $x^2 + xy - 6y^2 + 7x + 31y - 18 = 0$ and are at right angles to them.

Solution

$$\text{Given: } a = 1 \quad b = -6 \quad h = \frac{1}{2} \quad f = 1 \quad g = \frac{1}{2}$$

$$\begin{aligned} \text{Point of intersection} &= \left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2} \right) \\ &= \left(\frac{-\frac{5}{2}(1) - 4(\frac{1}{2})}{1 \times 4 - \frac{25}{4}}, \frac{\frac{1}{2} \times (-\frac{5}{2}) - 1(1)}{1 \times 4 - \frac{25}{4}} \right) \\ &= (2, 1) \end{aligned}$$

Substitute the values in the given formula

$$b(x - x_1)^2 - 2h(x - x_1)(y - y_1) + a(y - y_1)^2 = 0$$

$$4(x - 2)^2 + 5(x - 2)(y - 1) + 1(y - 1)^2 = 0$$

$$4(x^2 + 4 - 4x) + 5(xy - x - 2y + 2) + (y^2 + 1 - 2y) = 0$$

$$4x^2 + 16 - 16x + 5xy - 5x - 10y + 10 + y^2 + 1 - 2y = 0$$

$$4x^2 + 5xy + y^2 - 21x - 12y + 27 = 0$$