

Unit 1 Section 4 : Other Bases

The ideas that we have considered can be extended to other number bases.

The table lists the digits used in some other number bases.

Base	Digits Used
2	0, 1
3	0, 1, 2
4	0, 1, 2, 3
5	0, 1, 2, 3, 4

The powers of the base number give the place values when you convert to base 10. For example, for base 3, the place values are the powers of 3, i.e. 1, 3, 9, 27, 81, 243, etc.

This is shown in the following example, which also shows how the base 3 number 12100 is equivalent to the base 10 number 144.

Base 3

$$\begin{array}{r} 81 \quad 27 \quad 9 \quad 3 \quad 1 \\ \hline 1 \quad 2 \quad 1 \quad 0 \quad 0 \end{array} \rightarrow (1 \times 81) + (2 \times 27) + (1 \times 9) + (0 \times 3) + (0 \times 1) = 144 \text{ in base 10}$$

The following example shows a conversion from base 5 to base 10 using the powers of 5 as place values.

Base 5

$$\begin{array}{r} 625 \quad 125 \quad 25 \quad 5 \quad 1 \\ \hline 4 \quad 1 \quad 0 \quad 0 \quad 1 \end{array} \rightarrow \begin{array}{l} (4 \times 625) + (1 \times 125) + (0 \times 25) + (0 \times 5) + (1 \times 1) = 2626 \text{ in base} \\ 10 \end{array}$$

Example 1

Convert each of the following numbers to base 10:

- (a) 412 in base 6
- (b) 374 in base 9
- (c) 1432 in base 5

$$(4 \times 36) + (1 \times 6) + (2 \times 1) = 152$$

$$(3 \times 81) + (7 \times 9) + (4 \times 1) = 310$$

$$(1 \times 125) + (4 \times 25) + (3 \times 5) + (2 \times 1) = 242$$

Example 2

Convert each of the following base 10 numbers to the base stated:

(a) 472 to base 4

$$(1 \times 256) + (3 \times 64) + (1 \times 16) + (2 \times 4) + (0 \times 1) = 13120$$

(b) 179 to base 7

$$(3 \times 49) + (4 \times 7) + (4 \times 1) = 344$$

(c) 342 to base 3

$$(1 \times 243) + (1 \times 81) + (0 \times 27) + (2 \times 9) + (0 \times 3) + (0 \times 1) = 110200$$

Example 3

Carry out each of the following calculations in the base stated:

(a) $14 + 21$ base 5

$$40$$

(b) $16 + 32$ base 7

$$51$$

(c) $141 + 104$ base 5

$$300$$

(d) $212 + 121$ base 3

$$1110$$

Check your answer in (a) by changing to base 10 numbers

$$9 + 11 = 20$$

Example 4

Carry out each of the following multiplications in the base stated:

(a) 141×23 base 5

$$4343$$

(b) 122×12 base 3

$$10011$$

(c) 512×24 base 6

$$21532$$

Check your answer to (b) by converting to base 10 numbers

$$17 \times 5 = 85$$

Question 1

Convert the following numbers from the base stated to base 10:

(a) 412 base 5

(b) 333 base 4

(c) 728 base 9

(d) 1210 base 3

(e) 1471 base 8

(f) 612 base 7

(g) 351 base 6

(h) 111 base 3

Question 2

Convert the following numbers from base 10 to the base stated:

(a) 24 to base 3

(b) 16 to base 4

(c) 321 to base 5

(d) 113 to base 6

(e) 314 to base 7

(f) 84 to base 9

(g) 142 to base 3

(h) 617 to base 5

Question 3

Carry out the following additions in the base stated:

(a) $3 + 2$ in base 4

(b) $5 + 8$ in base 9

(c) $4 + 6$ in base 8

(d) $2 + 2$ in base 3

(e) $6 + 7$ in base 9

(f) $3 + 4$ in base 6

Question 4

In what number bases could each of the following numbers be written:

(a) 123 Base:

(b) 112 Base:

(c) 184 Base:

Question 5

Carry out each of the following calculations in the base stated:

(a) $13 + 23$ in base 4

(b) $120 + 314$ in base 5

(c) $222 + 102$ in base 3

(d) $310 + 132$ in base 4

(e) $624 + 136$ in base 7

(f) $211 + 142$ in base 5

(g) $333 + 323$ in base 4

(h) $141 + 424$ in base 5

Check your answers to parts (a), (c) and (e) by converting to base 10 numbers.

(a) + =

(c) + =

(e) + =

Question 6

Carry out each of the following multiplications in the base stated:

(a) 3×2 in base 4

(b) 4×3 in base 5

(c) 4×2 in base 6

(d) 3×5 in base 6

(e) 2×2 in base 3

(f) 8×8 in base 9

Question 7

Carry out each of the following multiplications in the base stated:

(a) 121×11 in base 3

(b) 133×12 in base 4

(c) 13×24 in base 5

(d) 142×14 in base 5

(e) 161×24 in base 7

(f) 472×32 in base 8

(g) 414×22 in base 5

(h) 2101×21 in base 3

Check your answers to parts (a), (c) and (e) by converting to base 10 numbers.

(a) $\times$ =

(c) $\times$ =

(e) $\times$ =

Question 8

In which base was each of the following calculations carried out?

(a) $4 + 2 = 11$ Base:

(b) $7 + 5 = 13$ Base:

(c) $8 \times 2 = 17$ Base:

(d) $4 \times 5 = 32$ Base:

(e) $11 - 3 = 5$ Base:

(f) $22 - 4 = 13$ Base:

Question 9

(a) Change 147 in base 8 into a base 3 number.

(b) Change 321 in base 4 into a base 7 number.

(c) Change 172 in base 9 into a base 4 number.

(d) Change 324 in base 5 into a base 6 number.

Question 10

In which base was each of the following calculations carried out?

(a) $171 \times 12 = 2272$ Base:

(b) $122 \times 21 = 11102$ Base:

(c) $24 \times 32 = 1252$ Base:

(d) $333 \times 33 = 23144$ Base: