RATIONAL EXPONENT

- $a^m \times a^n = a^{m+n}$ — To **multiply** numbers with the same base, keep the base and **add** the indices.
- $\dfrac{a^m}{a^n} = a^{m-n}$ — To **divide** numbers with the same base, keep the base and **subtract** the indices.
- $(a^m)^n = a^{m\times n}$ — When raising a power to a power, keep the base and **multiply** the indices.
- $(ab)^n = a^n b^n$ — The power of a product is the product of the powers.
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ — The power of a quotient is the quotient of the powers.
- $a^0 = 1, \quad a \neq 0$ — Any non-zero number raised to the power of zero is 1.
- $a^{-n} = \dfrac{1}{a^n}$ and $\dfrac{1}{a^{-n}} = a^n$ and in particular $a^{-1} = \dfrac{1}{a}$.

Notice that $(a^{\frac{1}{2}})^2 = a^{\frac{1}{2}\times 2} = a^1 = a$ and $(\sqrt{a})^2 = a$, so $a^{\frac{1}{2}} = \sqrt{a}$.

and $(a^{\frac{1}{3}})^3 = a^{\frac{1}{3}\times 3} = a^1 = a$ and $(\sqrt[3]{a})^3 = a$, so $a^{\frac{1}{3}} = \sqrt[3]{a}$.

In general, $a^{\frac{1}{n}} = \sqrt[n]{a}$ where $\sqrt[n]{a}$ is called the 'nth root of a'.

Example

Simplify: **a** $49^{\frac{1}{2}}$ **b** $27^{\frac{1}{3}}$ **c** $49^{-\frac{1}{2}}$ **d** $27^{-\frac{1}{3}}$

a $49^{\frac{1}{2}} = \sqrt{49} = 7$

b $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

c $49^{-\frac{1}{2}} = \dfrac{1}{49^{\frac{1}{2}}} = \dfrac{1}{\sqrt{49}} = \frac{1}{7}$

d $27^{-\frac{1}{3}} = \dfrac{1}{27^{\frac{1}{3}}} = \dfrac{1}{\sqrt[3]{27}} = \frac{1}{3}$

Evaluate without using a calculator:

a $4^{\frac{1}{2}}$ **b** $4^{-\frac{1}{2}}$ **c** $16^{\frac{1}{2}}$ **d** $16^{-\frac{1}{2}}$

e. $8^{\frac{1}{3}}$ f. $8^{-\frac{1}{3}}$ g. $64^{\frac{1}{3}}$ h. $64^{-\frac{1}{3}}$

Write the following in index form:

a $\sqrt{10}$ **b** $\frac{1}{\sqrt{10}}$ **c** $\sqrt[3]{15}$ **d** $\frac{1}{\sqrt[3]{15}}$

e $\sqrt[4]{19}$ **f** $\frac{1}{\sqrt[4]{19}}$ **g** $\sqrt[5]{13}$ **h** $\frac{1}{\sqrt[5]{13}}$

Example

Simplify: **a** $27^{\frac{4}{3}}$ **b** $16^{-\frac{3}{4}}$

a $27^{\frac{4}{3}} = (3^3)^{\frac{4}{3}}$
$= 3^4$
$= 81$

b $16^{-\frac{3}{4}} = (2^4)^{-\frac{3}{4}}$
$= 2^{-3}$
$= \frac{1}{2^3}$
$= \frac{1}{8}$

Without using a calculator, find the value of the following:

a $8^{\frac{4}{3}}$ **b** $8^{-\frac{2}{3}}$ **c** $4^{\frac{3}{2}}$ **d** $4^{-\frac{3}{2}}$ **e** $27^{\frac{2}{3}}$ **f** $27^{-\frac{2}{3}}$

g $32^{\frac{2}{5}}$ **h** $32^{-\frac{3}{5}}$ **i** $64^{\frac{5}{6}}$ **j** $125^{-\frac{2}{3}}$ **k** $81^{\frac{3}{4}}$ **l** $81^{-\frac{3}{4}}$