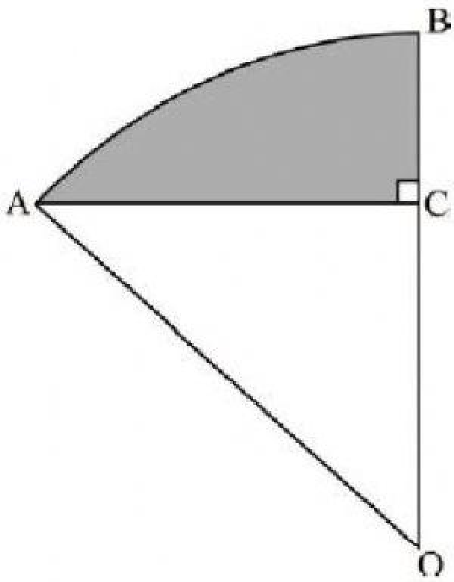
QUESTION 5

The diagram shows an arc, AB, of a circle with centre O and radius r. The line AC is drawn perpendicular to the line OCB. The region bounded by AC, BC and arc AB is shaded. $A\hat{O}B = \frac{\pi}{6}$ radians.



5.1 Find the area of ΔAOC, in terms of r. (6)

$; = \frac{1}{2}r^2\left(\frac{\pi}{6}\right)$

$= \frac{\pi r^2}{}$

5.3 If the shaded area is $\frac{2\pi-3\sqrt{3}}{6}$ cm^2, calculate the value of r. (6)

[16]

$= \frac{\pi r^2}{} - \frac{\sqrt{3}r^2}{} = \frac{2\pi-3\sqrt{3}}{}$

$\therefore 2\pi r^2 - 3\sqrt{3}r^2 = 8\pi - 12\sqrt{3}$

$\therefore r^2 = 4 \quad i.e.\ r = 2$

5.2 Find the area of the sector OAB, in terms of r. (4)

$= \frac{1}{2} r^2 \left(\frac{\pi}{6}\right)$

$= \frac{\pi r^2}{\ }$

Match the following rational functions to the appropriate graphs, A–F, below:

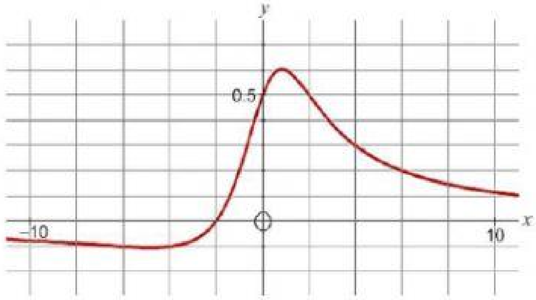
6.1 $f(x) = \frac{x^2-4}{x+2}$

6.2 $f(x) = \frac{x-2}{x^2-x-6}$

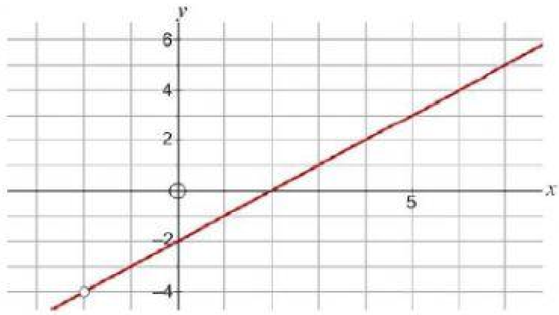
6.3 $f(x) = \frac{x^2+x-6}{x^2-6x+9}$

6.4 $f(x) = \frac{x^2-4x+4}{x-3}$

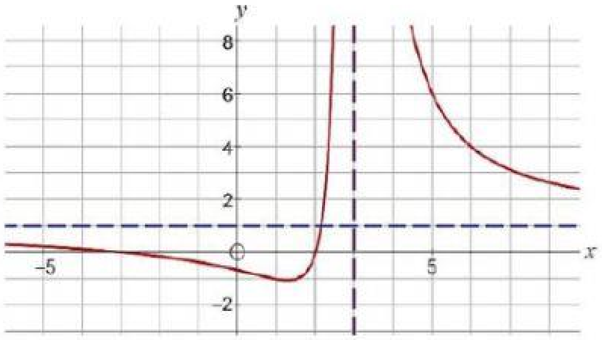
A



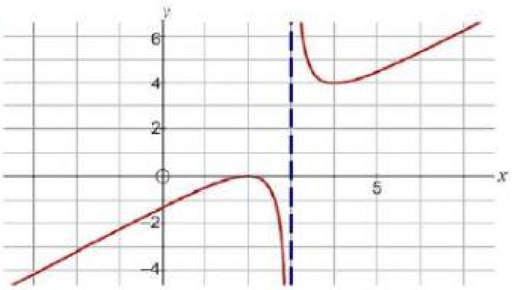
B



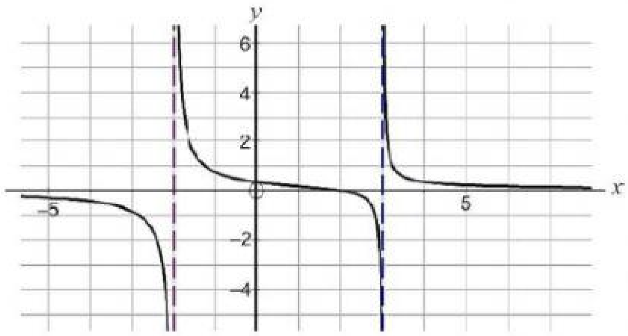
C



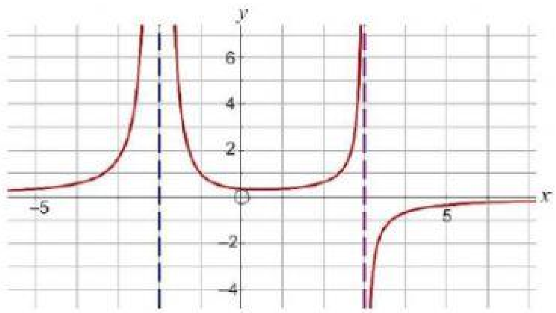
D



E



F



QUESTION 7

Find the equation of the tangent to the curve $x^3 - 2y^2 = 14 - 4x$ at the point (2; 1).

[11]

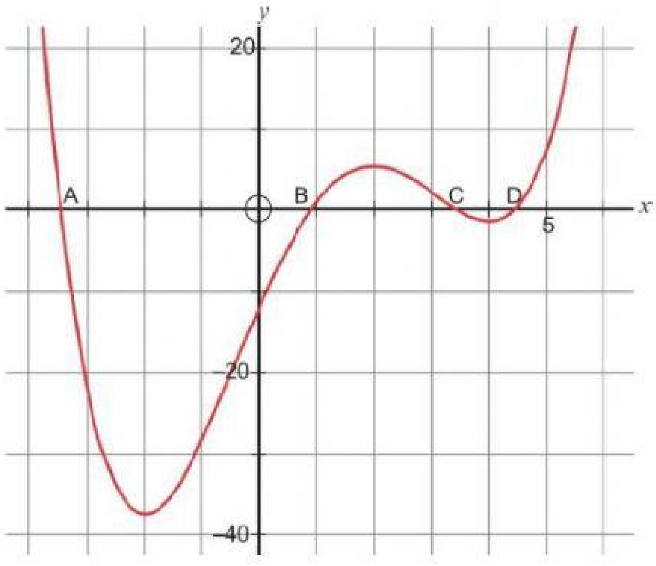
$$3x^2 - 4y \cdot \frac{dy}{dx} = -4$$

$$\therefore \frac{dy}{dx} = \frac{3x^2+4}{4y}$$

$$\therefore \frac{dy}{dx} = 4$$

QUESTION 8

The graph of $f(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 - 2x^2 + 16x - 12$ is shown, with stationary points at $x = -2$, $x = 2$ and $x = 4$. The graph has x-intercepts at the points indicated by A, B, C and D.



8.1 Without first solving the equation, state with clear justification which of the intercepts, A, B, C or D, will be found using Newton's method with an initial approximation of $x_0 = 2{,}1$. (3)

8.2 State any restrictions on the initial approximation of x. (2)

8.3 Given $x_0 = 3$, determine the x-intercept at C, correct to 6 decimal places. (8)

[13]

$$x_{r+1} = x_r - \frac{\frac{1}{4}x^4 - \frac{4}{3}x^3 - 2x^2 + 16x - 12}{x^3 - 4x^2 - 4x + 16}$$

x_0

x_1

$x_2 =$

$x_3 =$

QUESTION 9

A function is given as $f(x) = x + 4(x+1)^{-2}$

9.1 Determine the coordinates of the stationary point and prove that this is a local minimum. (10)

$f'(x) =$

$1 - \frac{8}{(x+1)^3}$

$\therefore x + 1 = 2$

$\therefore x = 1$ and

$f''(x) =$

$\therefore f''(1) = \frac{3}{2} > 0$