

GRADE 12 EXAMINATION

QUESTION 1

1.1 Solve for $x \in \mathbb{R}$ without the use of a calculator:

(a) $|x + 3| + 2x = 4$ (7)

$$x > -3 \quad x + 3 + 2x = 4$$

$$\therefore 3x = 1$$

$$\therefore x = \frac{1}{3}$$

n/a

(b) $\cos^{-1}\left(\frac{x^2}{8}\right) = \frac{\pi}{3}$ (4)

$$\frac{x^2}{8} = \frac{1}{2}$$

(c) $\ln x^2 - 3 \log_x e = 1$ (8)

$$2 \ln x - \frac{3}{\ln x} = 1$$

$$k = 1,5$$

$$k = -1$$

$$x = e^{1,5}$$

$$x = e^{-1}$$

1.2 Determine the domain and range of the graph of:

$$y = \ln(e^2 - x^2)$$

(8)
[27]

$e^2 - x^2$

$: \ln e^2 = 2$

QUESTION 2

2.1 Simplify: $\sqrt{i^4}$

(2)

i^4

$\sqrt{1}$

2.2 Find the real values of a and b such that $(3+2i)(a+3i) = bi$

(7)

$$(a+3i)(3+2i) = bi$$

Compare real:

$$a = 2$$

Compare imaginary: $2a + 9 = b$

2.3 One of the solutions to the equation $x^2 - 2x + p = 0$ is $x = q + \sqrt{3}i$.

Find the rational values of p and q .

(7)
[16]

$x = q - \sqrt{3}i$ is also a root

Sum of roots =

$= q^2 + 3 = p$

QUESTION 3

3.1 State whether each of the following statements is TRUE or FALSE:

- (a) If a function is differentiable at a point, then the limit of the function must exist at that point.
- (b) If a function is continuous at a point, then it must also be differentiable at that point.
- (c) If the limit of the function does not exist at a point, then the graph will have an asymptote at that point.
- (d) If the second derivative of a function at a point is equal to zero, then there will be a point of inflection on the graph at that point.
- (e) A function can exist at a point, whether or not the limit of the function exists at that point.
- (f) At a local maximum, the *gradient* of the graph is decreasing.

(12)

3.2 A function is defined as follows:

$$f(x) = \begin{cases} p - x^2 & \text{if } x \leq 2 \\ qx + 10 & \text{if } x > 2 \end{cases}$$

Calculate the value(s) of p and q if f is differentiable at $x = 2$.

(8)

Continuity:

$$\lim_{x \rightarrow 2^-} (p - x) = \lim_{x \rightarrow 2^+} (qx + 10)$$

Gradients equal:

$$\lim_{x \rightarrow 2^-} f'(x) = \lim_{x \rightarrow 2^+} f'(x)$$

$$q = -4$$

QUESTION 4

Alisha wants to prove by induction that $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Her teacher has taught her the following procedure:

Step 1: Prove true for $n = 1$

Step 2: Assume true for $n = k$

Step 3: Prove true for $n = k + 1$

Step 4: Conclude the proof.

Show Alisha's working for Step 3.

Prove true for $n = k + 1$

$$\frac{1}{\boxed{}} + \frac{1}{\boxed{}} + \frac{1}{\boxed{}} + \dots + \frac{1}{k(k+1)} + \frac{1}{\boxed{}} = \frac{k}{\boxed{}} + \frac{1}{\boxed{}}$$

$$\boxed{} = \frac{k(k+2)+1}{\boxed{}} = \frac{(k+1)^2}{\boxed{}} = \frac{k+1}{k+2} = \boxed{}$$

