

LKPD-05

MATRIKS

A. Simak video berikut



B. Menentukan invers suatu matriks

Misalkan:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

Invers matriks A adalah A^{-1}

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adjoint } A = \frac{1}{|A|} \cdot \text{Adj } A = \frac{1}{|A|} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Misalkan:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

Invers matriks A adalah A^{-1}

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adjoint } A = \frac{1}{|A|} \cdot \text{Adj } A$$

Akan dipelajari
mencari
Adjoin A

Pada matriks
berordo 3x3

Contoh:

1. Menentukan invers matriks $A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$
2. Menentukan invers matriks $E = \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix}$

Pembahasan:

1. $A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$

$$A^{-1} = \frac{1}{|A|} \cdot \text{Adj } A = \frac{1}{2 \cdot 3 - 5 \cdot 1} \begin{pmatrix} 3 & -1 \\ -5 & 1 \end{pmatrix} = 1 \begin{pmatrix} 3 & -1 \\ -5 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 1 \end{pmatrix}$$

Jadi

$$A^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & 1 \end{pmatrix}$$

2. $E = \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix}$

Menentukan terlebih dahulu determinan matriks E

$$\begin{aligned} |E| &= \begin{vmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{vmatrix} \\ &= (-1)(-1)(-2) + (0)(1)(5) + (3)(2)(1) \\ &\quad - (5)(-1)(3) - (1)(1)(-1) - (-2)(2)(0) \\ &= -2 + 0 + 6 + 15 + 1 - 0 \\ &= 20 \end{aligned}$$

Ingat tanda untuk mencari matriks kofaktor = K

Ingat tanda:

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$$

Membuat matriks K (Matriks Kofaktor)

Contoh elemen:

$$k_{11} = \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 2 - 1$$

$$= 1$$

$$E = \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix}$$

$$k_{21} = - \begin{vmatrix} 0 & 3 \\ 1 & -2 \end{vmatrix}$$

$$= -(0 - 3)$$

$$= -(-3)$$

$$= 3$$

$$E = \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix}$$

Ingat selalu tanda

Buat matriks kofaktor yaitu matriks K

$$K = \begin{pmatrix} + \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} & - \begin{vmatrix} 2 & 1 \\ 5 & -2 \end{vmatrix} & + \begin{vmatrix} 2 & -1 \\ 5 & 1 \end{vmatrix} \\ - \begin{vmatrix} 0 & 3 \\ 1 & -2 \end{vmatrix} & + \begin{vmatrix} -1 & 3 \\ 5 & -2 \end{vmatrix} & - \begin{vmatrix} -1 & 0 \\ 5 & 1 \end{vmatrix} \\ + \begin{vmatrix} 0 & 3 \\ -1 & 1 \end{vmatrix} & - \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix} & + \begin{vmatrix} -1 & 0 \\ 2 & -1 \end{vmatrix} \end{pmatrix}$$

$$K = \begin{pmatrix} (2-1) & -(-1-5) & (2-(-5)) \\ -(0-3) & (2-15) & -(-1-0) \\ (0-(-3)) & -(-1-6) & (1-0) \end{pmatrix}$$

$$K = \begin{pmatrix} 1 & 6 & 7 \\ 3 & -13 & 1 \\ 3 & 7 & 1 \end{pmatrix}$$

Sudah kalian temukan matriks kofaktor $K = \begin{pmatrix} 1 & 9 & 7 \\ 3 & -13 & 1 \\ 3 & 7 & 1 \end{pmatrix}$

$$\text{Adjoin } E = \text{Adj } E = K^T = \begin{pmatrix} 1 & 3 & 3 \\ 9 & -13 & 7 \\ 7 & 1 & 1 \end{pmatrix}$$

$$E^{-1} = \frac{1}{|E|} \cdot \text{Adj } E = \frac{1}{20} \begin{pmatrix} 1 & 3 & 3 \\ 9 & -13 & 7 \\ 7 & 1 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{20} & \frac{3}{20} & \frac{3}{20} \\ \frac{9}{20} & -\frac{13}{20} & \frac{7}{20} \\ \frac{7}{20} & \frac{1}{20} & \frac{1}{20} \end{pmatrix}$$

Jadi

$$E^{-1} = \frac{1}{20} \begin{pmatrix} 1 & 3 & 3 \\ 9 & -13 & 7 \\ 7 & 1 & 1 \end{pmatrix}$$

Atau ditulis:

$$E^{-1} = \begin{pmatrix} \frac{1}{20} & \frac{3}{20} & \frac{3}{20} \\ \frac{9}{20} & -\frac{13}{20} & \frac{7}{20} \\ \frac{7}{20} & \frac{1}{20} & \frac{1}{20} \end{pmatrix}$$

Kalian bisa mengecek, apakah invers matriks yang kalian temukan itu benar atau tidak, dengan cara:

$$\text{Bila } E \cdot E^{-1} = E^{-1} \cdot E = I$$

Cek:

$$E \cdot E^{-1}$$

$$= \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix} \begin{pmatrix} \frac{1}{20} & \frac{3}{20} & \frac{3}{20} \\ \frac{9}{20} & -\frac{13}{20} & \frac{7}{20} \\ \frac{7}{20} & \frac{1}{20} & \frac{1}{20} \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix} \cdot \frac{1}{20} \begin{pmatrix} 1 & 3 & 3 \\ 9 & -13 & 7 \\ 7 & 1 & 1 \end{pmatrix}$$

$$= \frac{1}{20} \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 3 & 3 \\ 9 & -13 & 7 \\ 7 & 1 & 1 \end{pmatrix}$$

$$= \frac{1}{20} \begin{pmatrix} -1.1 + 0.9 + 3.7 & -1.3 + 0.(-13) + 3.1 & -1.3 + 0.7 + 3.1 \\ 2.1 + (-1).9 + 1.7 & 2.3 + (-1)(-13) + 1.1 & 2.3 + (-1).7 + 1.1 \\ 5.1 + 1.9 + (-2).7 & 5.3 + 1.(-13) + (-2).1 & 5.3 + 1.7 + (-2).1 \end{pmatrix}$$

$$= \frac{1}{20} \begin{pmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

C. Pilih benar atau salah dari pernyataan berikut.

No	Pernyataan	Benar/ Salah
1.	Invers matriks $P = \begin{pmatrix} 2 & 1 \\ 4 & 6 \end{pmatrix}$ adalah $P^{-1} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{8} \\ -\frac{1}{2} & \frac{1}{4} \end{pmatrix}$	
2.	Invers matriks $T = \begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}$ adalah $T^{-1} = \begin{pmatrix} \frac{5}{7} & \frac{1}{7} \\ -\frac{3}{7} & \frac{2}{7} \end{pmatrix}$	
3.	Invers matriks $H = \begin{pmatrix} 6 & 0 \\ 4 & 2 \end{pmatrix}$ adalah $H^{-1} = \frac{1}{12} \begin{pmatrix} 2 & 0 \\ -4 & 6 \end{pmatrix}$	
4.	Invers matriks $A = \begin{pmatrix} -2 & 4 & -5 \\ 1 & 3 & -7 \\ 0 & 4 & -8 \end{pmatrix}$ adalah $A^{-1} = \frac{1}{4} \begin{pmatrix} 4 & 12 & -13 \\ 8 & 16 & -19 \\ 4 & 8 & -10 \end{pmatrix}$	

Matriks $A = \begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix}$ tidak memiliki invers.

Mengapa?

----Selamat Belajar, setelah ini lanjut LKPD 06----