

LKPD-04

MATRIKS

A. Simak video berikut



B. Perhatikan kembali matriks-matriks berikut. (Matriks yang tersaji di LKPD 03)

$A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$	$B = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$	$C = \begin{pmatrix} 2 & -1 \\ 0 & 6 \end{pmatrix}$	$D = \begin{pmatrix} \frac{6}{12} & \frac{1}{12} \\ 0 & \frac{2}{12} \end{pmatrix}$
$E = \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix}$	$F = \begin{pmatrix} \frac{1}{20} & \frac{3}{20} & \frac{3}{20} \\ \frac{9}{20} & -\frac{13}{20} & \frac{7}{20} \\ \frac{7}{20} & \frac{1}{20} & \frac{1}{20} \end{pmatrix}$	$G = \begin{pmatrix} 0 & 4 & 1 \\ 3 & 5 & -2 \\ 0 & 3 & 6 \end{pmatrix}$	$H = \begin{pmatrix} -\frac{36}{63} & \frac{21}{63} & \frac{13}{63} \\ \frac{18}{63} & 0 & -\frac{3}{63} \\ -\frac{9}{63} & 0 & \frac{12}{63} \end{pmatrix}$

1. Matriks hasil $A \times B$ adalah

$A \times B$

$$\begin{aligned}
 &= \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} \\
 &= \begin{pmatrix} 2.3 + 1.(-5) & 2.(-1) + 1.2 \\ 5.3 + 3.(-5) & 5.(-1) + 3.2 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I
 \end{aligned}$$

2. Matriks hasil $C \times D$ adalah

$C \times D$

$$\begin{aligned}
 &= \begin{pmatrix} 2 & -1 \\ 0 & 6 \end{pmatrix} \begin{pmatrix} \frac{6}{12} & \frac{1}{12} \\ 0 & \frac{2}{12} \end{pmatrix} \\
 &= \begin{pmatrix} 2 & -1 \\ 0 & 6 \end{pmatrix} \cdot \frac{1}{12} \begin{pmatrix} 6 & 1 \\ 0 & 2 \end{pmatrix} \\
 &= \frac{1}{12} \begin{pmatrix} 2 & -1 \\ 0 & 6 \end{pmatrix} \begin{pmatrix} 6 & 1 \\ 0 & 2 \end{pmatrix} \\
 &= \frac{1}{12} \begin{pmatrix} 2.6 + (-1).0 & 2.1 + (-1).2 \\ 0.6 + 6.0 & 0.6 + 6.2 \end{pmatrix} \\
 &= \frac{1}{12} \begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I
 \end{aligned}$$

3. Matriks hasil $E \times F$ adalah

$E \times F$

$$\begin{aligned}
 &= \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix} \begin{pmatrix} \frac{1}{20} & \frac{3}{20} & \frac{3}{20} \\ \frac{9}{20} & -\frac{13}{20} & \frac{7}{20} \\ \frac{7}{20} & \frac{1}{20} & \frac{1}{20} \end{pmatrix} \\
 &= \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix} \cdot \frac{1}{20} \begin{pmatrix} 1 & 3 & 3 \\ 9 & -13 & 7 \\ 7 & 1 & 1 \end{pmatrix} \\
 &= \frac{1}{20} \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 3 & 3 \\ 9 & -13 & 7 \\ 7 & 1 & 1 \end{pmatrix} \\
 &= \frac{1}{20} \begin{pmatrix} -1.1 + 0.9 + 3.7 & -1.3 + 0.(-13) + 3.1 & -1.3 + 0.7 + 3.1 \\ 2.1 + (-1).9 + 1.7 & 2.3 + (-1)(-13) + 1.1 & 2.3 + (-1).7 + 1.1 \\ 5.1 + 1.9 + (-2).7 & 5.3 + 1.(-13) + (-2).1 & 5.3 + 1.7 + (-2).1 \end{pmatrix} \\
 &= \frac{1}{20} \begin{pmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I
 \end{aligned}$$

Ingat:

I adalah matriks identitas

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

4. Matriks hasil $G \times H$ adalah

$G \times H$

$$\begin{aligned}
 &= \begin{pmatrix} 0 & 4 & 1 \\ 3 & 5 & -2 \\ 0 & 3 & 6 \end{pmatrix} \begin{pmatrix} -\frac{36}{63} & \frac{21}{63} & \frac{13}{63} \\ \frac{18}{63} & 0 & -\frac{3}{63} \\ \frac{9}{63} & 0 & \frac{12}{63} \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 4 & 1 \\ 3 & 5 & -2 \\ 0 & 3 & 6 \end{pmatrix} \cdot \frac{1}{36} \begin{pmatrix} -36 & 21 & 13 \\ 18 & 0 & -3 \\ -9 & 0 & 12 \end{pmatrix} \\
 &= \frac{1}{36} \begin{pmatrix} 0 & 4 & 1 \\ 3 & 5 & -2 \\ 0 & 3 & 6 \end{pmatrix} \begin{pmatrix} -36 & 21 & 13 \\ 18 & 0 & -3 \\ -9 & 0 & 12 \end{pmatrix} \\
 &= \frac{1}{36} \begin{pmatrix} 0.(-36) + 4.18 + 1.(-9) & 0.21 + 4.0 + 1.0 & 0.13 + 4.(-3) + 1.12 \\ 3.(-36) + 5.18 + (-2).(-9) & 3.21 + 5.0 + (-2).0 & 3.13 + 5.(-3) + (-2).12 \\ 0.(-36) + 3.18 + 6.(-9) & 0.21 + 3.0 + 6.0 & 0.13 + 3.(-3) + 6.12 \end{pmatrix} \\
 &= \frac{1}{36} \begin{pmatrix} 63 & 0 & 0 \\ 0 & 63 & 0 \\ 0 & 0 & 63 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I
 \end{aligned}$$

C. Kesimpulan 01

$$A \times B = I$$

B merupakan invers matriks A
Invers Matriks A dilambangkan A^{-1}
Sehingga $A \cdot A^{-1} = I$

Sebaliknya

$$A \times B = I$$

A merupakan invers matriks B
Invers Matriks B dilambangkan B^{-1}
Sehingga $B^{-1} \cdot B = I$

D. Kesimpulan 02

$$1) \quad A \times B = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I, \text{ maka } A^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}$$

$$2) \quad C \times D = \begin{pmatrix} 2 & -1 \\ 0 & 6 \end{pmatrix} \begin{pmatrix} \frac{6}{12} & \frac{1}{12} \\ 0 & \frac{2}{12} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I, \text{ maka } C^{-1} = \begin{pmatrix} \frac{6}{12} & \frac{1}{12} \\ 0 & \frac{2}{12} \end{pmatrix} = \frac{1}{12} \begin{pmatrix} 6 & 1 \\ 0 & 2 \end{pmatrix}$$

$$3) \quad E \times F = \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix} \begin{pmatrix} \frac{1}{20} & \frac{3}{20} & \frac{3}{20} \\ \frac{9}{20} & -\frac{13}{20} & \frac{7}{20} \\ \frac{7}{20} & \frac{1}{20} & \frac{1}{20} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$\text{Maka } E^{-1} = \begin{pmatrix} \frac{1}{20} & \frac{3}{20} & \frac{3}{20} \\ \frac{9}{20} & -\frac{13}{20} & \frac{7}{20} \\ \frac{7}{20} & \frac{1}{20} & \frac{1}{20} \end{pmatrix} = \frac{1}{20} \begin{pmatrix} 1 & 3 & 3 \\ 9 & -13 & 7 \\ 7 & 1 & 1 \end{pmatrix}$$

$$4) \quad G \times H = \begin{pmatrix} 0 & 4 & 1 \\ 3 & 5 & -2 \\ 0 & 3 & 6 \end{pmatrix} \begin{pmatrix} -\frac{36}{63} & \frac{21}{63} & \frac{13}{63} \\ \frac{18}{63} & 0 & -\frac{3}{63} \\ -\frac{9}{63} & 0 & \frac{12}{63} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$\text{Maka } G^{-1} = \begin{pmatrix} -\frac{36}{63} & \frac{21}{63} & \frac{13}{63} \\ \frac{18}{63} & 0 & -\frac{3}{63} \\ -\frac{9}{63} & 0 & \frac{12}{63} \end{pmatrix} = \frac{1}{36} \begin{pmatrix} -36 & 21 & 13 \\ 18 & 0 & -3 \\ -9 & 0 & 12 \end{pmatrix}$$

Bagaimana mencari invers suatu matriks?

1. Determinan Matriks

2. Invers Matriks

Determinan Matriks

E. Memahami determinan matriks

➤ Determinan matriks berordo 2x2

Determinan matriks A ditulis $\det A$ atau $|A|$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \text{ maka } \det A = |A| = ad - bc$$

Contoh:

- 1) Determinan matriks $P = \begin{pmatrix} 3 & -1 \\ 2 & 3 \end{pmatrix}$ adalah $|P| = 3.3 - 2.(-1) = 9 + 2 = 11$
- 2) Determinan matriks $H = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$ adalah $|H| = 2.2 - 4.1 = 4 - 4 = 0$

➤ Determinan matriks berordo 3x3

Determinan matriks A ditulis $\det A$ atau $|A|$

Menentukan determinan matriks berordo 3x3

1. Cara Sarrus

2. Cara Ekspansi Kofaktor

Contoh:

Menentukan invers matriks $E = \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix}$

Cara 1: Cara Sarrus

$$\begin{aligned} |E| &= \begin{vmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{vmatrix} \\ &= \begin{vmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{vmatrix} \begin{matrix} -1 & 0 \\ 2 & -1 \\ 5 & 1 \end{matrix} \\ &\quad \begin{matrix} - & - & - & + & + & + \end{matrix} \\ &= (-1)(-1)(-2) + (0)(1)(5) + (3)(2)(1) \\ &\quad - (5)(-1)(3) - (1)(1)(-1) - (-2)(2)(0) \\ &= -2 + 0 + 6 + 15 + 1 - 0 \\ &= 20 \end{aligned}$$

Cara 2: Cara Ekspansi Kofaktor

$$\begin{aligned} \text{Tanda :} \\ \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \\ E &= \begin{pmatrix} -1 & 0 & 3 \\ 2 & -1 & 1 \\ 5 & 1 & -2 \end{pmatrix} \\ |E| &= -1 \begin{vmatrix} -1 & 1 \\ 1 & -2 \end{vmatrix} - 0 \begin{vmatrix} 2 & 1 \\ 5 & -2 \end{vmatrix} + 3 \begin{vmatrix} 2 & -1 \\ 5 & 1 \end{vmatrix} \\ &= -1(2-1) - 0 + 3(2-(-5)) \\ &= -1 + 3 \cdot 7 \\ &= -1 + 21 = 20 \end{aligned}$$

F. Latihan Soal (Pilih jawaban dengan memilih di dalam kotak)

1. Determinan matriks $A = \begin{pmatrix} 2 & 5 \\ -2 & 3 \end{pmatrix}$ adalah
2. Determinan matriks $P = \frac{1}{2} \begin{pmatrix} 16 & -4 \\ 2 & 2 \end{pmatrix}$ adalah
3. Determinan matriks $Q = \begin{pmatrix} -3 & 1 \\ x & 2 \end{pmatrix}$ adalah 4, maka nilai x adalah
4. Determinan matriks $H = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 4 \\ 1 & -1 & 2 \end{pmatrix}$ adalah
5. Determinan matriks $U = \begin{pmatrix} -1 & 0 & 0 \\ 12 & 1 & 1 \\ 0 & 2 & 4 \end{pmatrix}$ adalah
6. Determinan matriks $T = \begin{pmatrix} 4 & 1 & 0 \\ 2 & 0 & -1 \\ 2 & 1 & 1 \end{pmatrix}$ adalah

----Selamat Belajar, setelah ini lanjut LKPD 05----