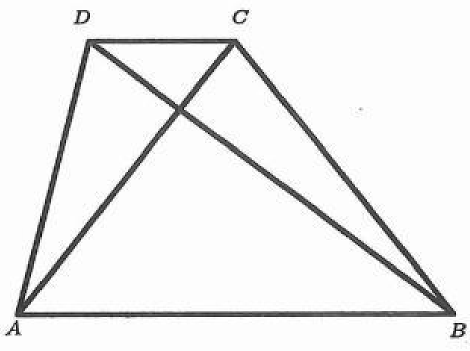
1. AREA

Examples

Example 1. The diagonals of a trapezium divide it into four triangles. Three of the areas of the triangles are equal to respectively 4, 6 and 9 sq.cm. What is the area of the trapezium?



Solution. Let $ABCD$ be the trapezium and O be the intersection of its diagonals, $AB > CD$.
The areas of the triangles ADO and BCO are equal, so the possible areas of the four triangles are $(4, 4, 6, 9)$; $(4, 6, 6, 9)$; $(4, 6, 9, 9)$.
Besides,

$$S_{ABO} > S_{DCO}, \frac{AO}{OC} = \frac{S_{AOD}}{S_{COD}} = \frac{S_{AOB}}{S_{COB}},$$

so the area of the areas of the triangles are $4, 6, 6$ and 9. Therefore, the area of the trapezium is $6 + 6 + 9 + 4 = 25$.

Example 2. In trapezoid ($AB > CD, AB > CD$), a is the length of its leg AD and b is the distance from the midpoint of AB to the leg AD (see the diagram). If $\frac{AB}{CD} = \frac{3}{2}$, express the area of the trapezoid in terms of a and b.

Solution. M is the midpoint of AB so the area of the triangle AMD is $\frac{ab}{2}$.
DM is a median in the triangle ABD, so the area of the triangle BMD is $\frac{ab}{2}$, which leads to the area of the triangle ABD is ab.
The areas of the triangles BCD and ABD are in the same ratio as AB and CD, so the area of the triangle ABD would be $\frac{2}{3}ab$.
Therefore, the area of the trapezium $ABCD$ is $\frac{5}{3}ab$.

Example 3. The lengths of the medians of a triangle are 9, 12 and 15 cm. Calculate the area of the triangle.

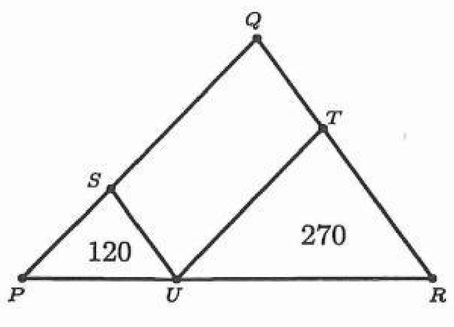
Solution. The lengths of the medians of the triangle ABC are, respectively, 9 cm, 12 cm and 15 cm. The point M is the centroid of the triangle.
Duplicate the triangle to get the parallelogram $ACBD$, where the point N is the centroid of the triangle ABD. Then,

$$AM = BN = 6, BM = 8, MN = 10.$$

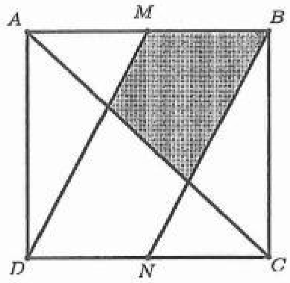
BMN is a right-angled triangle, and its area is $\frac{1}{3}$ of the area of the triangle ABC.
Therefore, the area of the triangle ABC is 72 sq. cm.

Practice

Problem 1. In $\triangle PQR$, U is a point on PR, S is a point on PQ, T is a point on QR with $US \parallel RQ$ and $UT \parallel PQ$. The area of $\triangle PSU$ is 120 cm^2 and the area of $\triangle TUR$ is 270 cm^2. What is the area of $\triangle QST$?



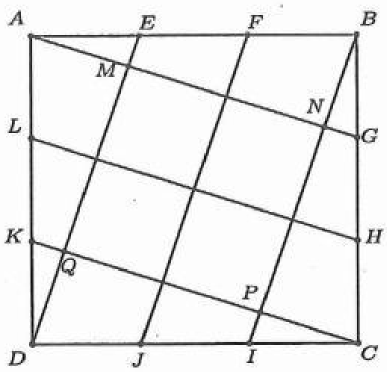
Problem 2. In the figure, $ABCD$ is a unit square, and M and N are the midpoints of sides $\overline{AB}$ and $\overline{CD}$, respectively. Find the area of the shaded region.



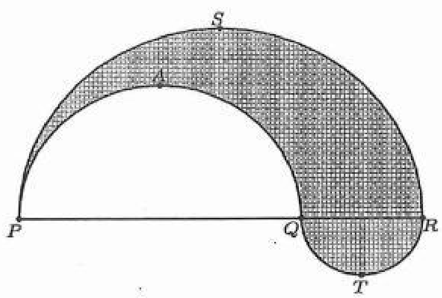
Problem 3. The diagram below shows square $ABCD$ such that

$$AE = EF = FB = BG = GH = HC = CI = IJ = JD = DK = KL = LA = 20cm$$

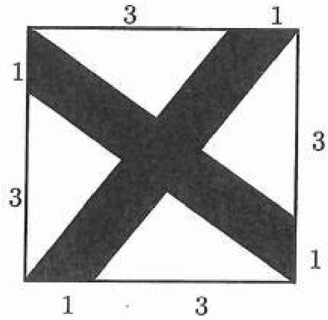
Find the area of the square $MNPQ$ in cm^2.



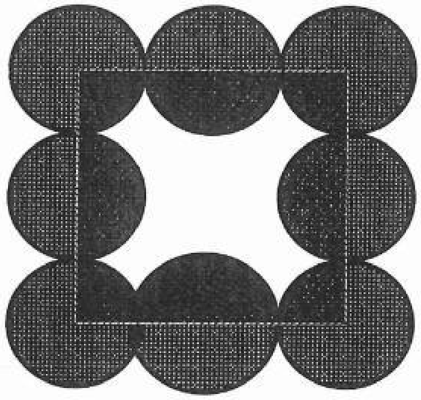
Problem 4. In the figure below, PSR, RTQ and PAQ are three semicircles of diameters 10 cm, 3 cm and 7 cm respectively. Find the area of the shaded region.



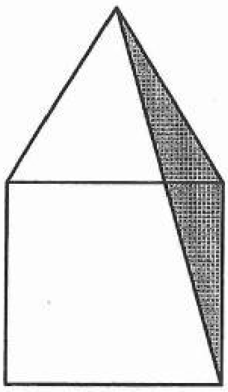
Problem 5. Given a square with side 4, find the area of the shaded region.



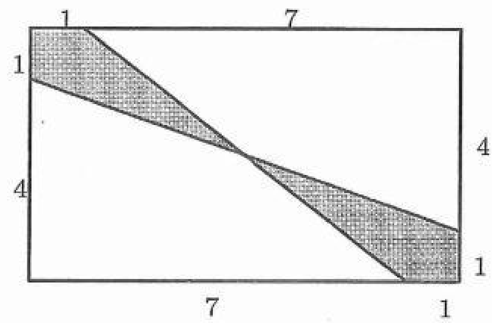
Problem 6. The shaded region is bounded by eight equal circles with centres at the corners and midpoints of the sides of a square.
The perimeter of the square is 8cm. What is the perimeter of the shaded region?



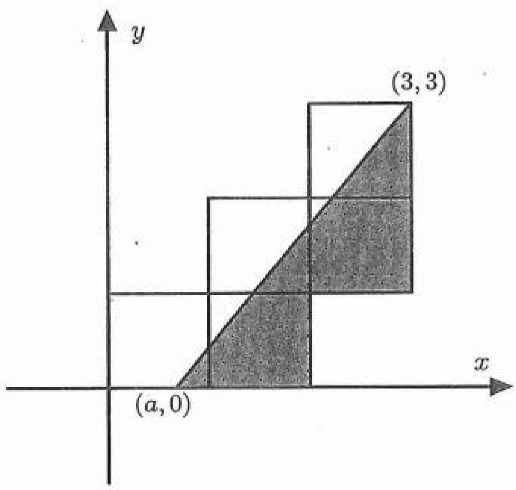
Problem 7. The below diagram shows an equilateral triangle and a square of side length 2 joined along an edge. What is the area of the shaded triangle?



Problem 8. What is the area of the shaded region of the given 8×5 rectangle?



Problem 9. Five unit squares are arranged in the coordinate plane as shown, with the lower left corner at the origin. The slanted line, extending from $(a, 0)$ to $(3, 3)$, divides the 5-square shape into two regions of equal area. What is the value of $3a$?



Problem 10. The diagram shows three touching circles each of radius 5cm. and a line which touches two of them.
Find the total length of the perimeter of the two shaded shapes.