1.1 Factors, multiples and primes

In this section you will ...

- write a positive integer as a product of prime factors
- use prime factors to find a highest common factor (HCF) and a lowest common multiple (LCM).

Key words

factor tree
highest common factor (HCF)
index
integer
lowest common multiple (LCM)
prime factor

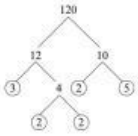
Any **integer** bigger than 1:

- is a prime number, or
- can be written as a product of prime numbers.

Example:

$46 = 2 \times 23$ 47 is prime $48 = 2 \times 2 \times 2 \times 2 \times 3$
$49 = 7 \times 7$ $50 = 2 \times 5 \times 5$

You can use a **factor tree** to write an integer as a product of its **prime factors**.



This is how to draw a factor tree for 120.

1 Write 120.
2 Draw branches to two numbers that have a product of 120. Do not use 1 as one of the numbers. Here we have chosen 12 and 10. $120 = 12 \times 10$
3 Do the same with 12 and 10. Here $12 = 3 \times 4$ and $10 = 2 \times 5$
4 3, 2 and 5 are prime numbers, so circle them.
5 Draw two more branches from 4. $4 = 2 \times 2$. Circle the 2s.
6 Now all the end numbers are prime, so stop.
7 120 is the product of all the end numbers: $120 = 2 \times 2 \times 2 \times 3 \times 5$
8 You can check that this is correct using a calculator.

You can also write the result like this: $120 = 2^3 \times 3 \times 5$

2^3 means $2 \times 2 \times 2$ and the small 3 is an **index**.

Now check that $75 = 3 \times 5^2$

You can use products of prime factors to find the **HCF** and **LCM** of two numbers.

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Worked example 1.1

a Find the LCM of 120 and 75.
b Find the HCF of 120 and 75.

Answer

a Write 120 and 75 as products of their prime factors:
$120 = \boxed{2 \times 2 \times 2 \times 3} \times 5$
$75 = 3 \times \boxed{5 \times 5}$
Look at the prime factors of both numbers.
For the LCM, use the **larger** frequency of each prime factor.
- 120 has three 2s and 75 has no 2s. The LCM must have three 2s.
- 120 has one 3 and 75 has one 3. The LCM must have one 3.
- 120 has one 5 and 75 has two 5s. The LCM must have two 5s.

The LCM is $2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^3 \times 3 \times 5^2 = 8 \times 3 \times 25 = 600$

b For the HCF use the **smaller** frequency of each factor: there are no 2s in 75, and there is one 3 and one 5 in both numbers.
Multiply these factors.
The HCF is $3 \times 5 = 15$

Exercise 1.1

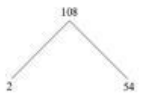
Think like a mathematician

1 The factor tree for 120 in Section 1.1 started with 12×10.



a Draw a factor tree for 120 that starts with 6×20.
b Compare your answer to part **a** with a partner's. Are your trees the same or different?
c Draw some different factor trees for 120. Can you say how many different trees are possible?
d Do all factor trees for 120 have the same end points?

2 **a** Complete this factor tree for 108.



b Draw a different factor tree for 108.
c Write 108 as a product of its prime factors.
d Compare your factor trees and your product of prime factors with a partner's. Have you drawn the same trees or different ones? Are your trees correct?

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3 a Draw a factor tree for 200 that starts with 10×20.
b Write 200 as a product of prime numbers.
c Compare your factor tree with a partner's. Have you drawn the same tree or different ones? Are your trees correct?
d How many different factor trees can you draw for 200 that start with 10×20?

4 a Draw a factor tree for 330.
b Write 330 as a product of prime numbers.

5 Match each number to a product of prime factors.
The first one has been done for you: **a** and **i**.

a	20	i	$2^2 \times 5$
b	24	ii	$2 \times 3 \times 7$
c	42	iii	$2^2 \times 3^2 \times 5$
d	50	iv	2×5^2
e	180	v	$2^3 \times 3$

6 Work out the product of each set of prime factors.
a $3^3 \times 5 \times 7$ b $2^3 \times 5^2$ c $2^2 \times 3^2 \times 11$
d $2^4 \times 7^2$ e 3×17^2

Tip
You can use a factor tree to help you.

7 Write each of these numbers as a product of prime factors.
a 28 b 60 c 72
d 153 e 190 f 275

8 a Copy the table and write each number as a product of prime numbers.

Number	Product of prime numbers
35	5×7
70	
140	
280	

b Add more rows to the table to continue the pattern.

9 a Write 1001 as a product of prime numbers.
b Write 4004 as a product of prime numbers.
c Write 6006 as a product of prime numbers.

10 a Use a factor tree to write 132 as a product of prime numbers.
b Write 150 as a product of prime numbers.
c $132 \times 150 = 19\,800$. Use this fact to write 19 800 as a product of prime numbers.

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11 **a** Write each of these numbers as a product of prime numbers.
i 15 **ii** 15^2 **iii** 28
iv 28^2 **v** 36 **vi** 36^2
b What do you notice about your answers to **i** and **ii**, **iii** and **iv**, **v** and **vi**?
c If $96 = 2^5 \times 3$, show how to find the prime factors of 96^2. Will your method work for all numbers?

12 $40 = 2 \times 2 \times 2 \times 5$ and $28 = 2 \times 2 \times 7$
Use these facts to find
a the HCF of 40 and 28 **b** the LCM of 40 and 28.

13 $450 = 2 \times 3 \times 3 \times 5 \times 5$ and $60 = 2 \times 2 \times 3 \times 5$
Use these facts to find
a the HCF of 450 and 60 **b** the LCM of 450 and 60.

14 $180 = 2^2 \times 3^2 \times 5$ and $54 = 2 \times 3^3$
Use these facts to find
a the HCF of 180 and 54 **b** the LCM of 180 and 54.

15 **a** Write 45 as a product of prime numbers.
b Write 75 as a product of prime numbers.
c Find the LCM of 45 and 75.
d Find the HCF of 45 and 75.

16 **a** Draw factor trees to find the LCM of 90 and 140.
b Compare your answer with a partner's. Did you draw the same factor trees? Have you both got the same answer?

17 **a** Write 396 as a product of prime numbers.
b Write 168 as a product of prime numbers.
c Find the HCF of 396 and 168.
d Find the LCM of 396 and 168.

18 **a** Find the HCF of 34 and 58.
b Find the LCM of 34 and 58.

19 Show that the HCF of 63 and 110 is 1.

20 37 and 47 are prime numbers.
a What is the HCF of 37 and 47?
b What is the LCM of 37 and 47?
c Write a rule for finding the HCF and LCM of two prime numbers.
d Compare your answer to part **c** with a partner's answer. Check your rules by finding the HCF and LCM of 39 and 83.

Tip
Use a calculator to help you.

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