
Contoh 1 (Dasar)

Hitunglah: $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

Pembahasan:

1. Substitusi: $\frac{\sqrt{4} - 2}{4 - 4} = \frac{0}{0}$ (Tak tentu).
2. Kali Sekawan: Kalikan dengan $\frac{\sqrt{x} + 2}{\sqrt{x} + 2}$.
$$\begin{aligned}\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2} \\&= \lim_{x \rightarrow 4} \frac{(\sqrt{x})^2 - 2^2}{(x - 4)(\sqrt{x} + 2)} \\&= \lim_{x \rightarrow 4} \frac{x - 4}{(x - 4)(\sqrt{x} + 2)}\end{aligned}$$
3. Sederhanakan: Coret $(x - 4)$.
$$\lim_{x \rightarrow 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{\sqrt{4} + 2} = \frac{1}{4}$$

Jawaban: $\frac{1}{4}$

Contoh 2 (Akar di Pembilang)

Hitunglah: $\lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x - 3}$

Pembahasan:

1. Substitusi: $\frac{\sqrt{4} - 2}{3 - 3} = \frac{0}{0}$.
2. Kali Sekawan: Sekawan dari $(\sqrt{x+1} - 2)$ adalah $(\sqrt{x+1} + 2)$.
$$\begin{aligned}\lim_{x \rightarrow 3} \frac{(\sqrt{x+1} - 2)(\sqrt{x+1} + 2)}{(x - 3)(\sqrt{x+1} + 2)} \\&= \lim_{x \rightarrow 3} \frac{x - 3}{(x - 3)(\sqrt{x+1} + 2)}\end{aligned}$$
3. Hasil: $\frac{1}{\sqrt{3+1} + 2} = \frac{1}{4}$

Jawaban: $\frac{1}{4}$

Contoh 3 (Akar di Penyebut)

Hitunglah: $\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+x} - 1}$

Pembahasan:

1. Kali Sekawan Penyebut: Kalikan dengan $(\sqrt{1+x} + 1)$.

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + 1)}{(1+x) - 1} \\ = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+x} + 1)}{x} \end{aligned}$$

2. Hasil: $\sqrt{1+0} + 1 = 1 + 1 = 2$

Jawaban: 2

Contoh 4 (Akar dengan Koefisien)

Hitunglah: $\lim_{x \rightarrow 2} \frac{\sqrt{2x+5} - 3}{x - 2}$

Pembahasan:

1. Kali Sekawan: $\frac{\sqrt{2x+5} + 3}{\sqrt{2x+5} + 3}$.

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{(2x+5) - 9}{(x-2)(\sqrt{2x+5} + 3)} \\ = \lim_{x \rightarrow 2} \frac{2x - 4}{(x-2)(\sqrt{2x+5} + 3)} \end{aligned}$$

2. Faktorkan: $2x - 4 = 2(x - 2)$.

$$\lim_{x \rightarrow 2} \frac{2(x-2)}{(x-2)(\sqrt{2x+5} + 3)} = \frac{2}{\sqrt{9} + 3} = \frac{2}{6} = \frac{1}{3}$$

Jawaban: $\frac{1}{3}$

Contoh 5 (Dua Bentuk Akar)

Hitunglah: $\lim_{x \rightarrow 1} \frac{\sqrt{2x-1} - \sqrt{x}}{x-1}$

Pembahasan:

1. Kali Sekawan: $(\sqrt{2x-1} + \sqrt{x})$.
$$\lim_{x \rightarrow 1} \frac{(2x-1) - (x)}{(x-1)(\sqrt{2x-1} + \sqrt{x})}$$
$$= \lim_{x \rightarrow 1} \frac{x-1}{(x-1)(\sqrt{2x-1} + \sqrt{x})}$$
2. Hasil: $\frac{1}{\sqrt{2(1)-1} + \sqrt{1}} = \frac{1}{1+1} = \frac{1}{2}$

Jawaban: $\frac{1}{2}$

Latihan Soal: Limit Fungsi Aljabar (Perkalian Sekawan)

Selesaikan soal-soal berikut secara bertahap. Pastikan untuk selalu mencoba metode substitusi terlebih dahulu!

Level 1: Mudah (Dasar Perkalian Sekawan)

1. $\lim_{x \rightarrow 9} \frac{x-9}{\sqrt{x}-3}$
2. $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$
3. $\lim_{x \rightarrow 0} \frac{\sqrt{x+4}-2}{x}$
4. $\lim_{x \rightarrow 2} \frac{x-2}{\sqrt{x+7}-3}$

$$5. \lim_{x \rightarrow 5} \frac{\sqrt{x-1} - 2}{x-5}$$

$$6. \lim_{x \rightarrow 0} \frac{3x}{\sqrt{9+x} - 3}$$

$$7. \lim_{x \rightarrow 1} \frac{1-x}{1-\sqrt{x}}$$

Level 2: Sedang (Melibatkan Pemfaktoran/Koefisien)

$$8. \lim_{x \rightarrow 4} \frac{\sqrt{2x+1} - 3}{x-4}$$

$$9. \lim_{x \rightarrow 1} \frac{\sqrt{3x+1} - 2}{x^2 - 1}$$

$$10. \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x+2} - 2}$$

$$11. \lim_{x \rightarrow -1} \frac{x+1}{\sqrt{x+5}-2}$$

$$12. \lim_{x \rightarrow 3} \frac{2-\sqrt{x+1}}{x-3}$$

$$13. \lim_{x \rightarrow 0} \frac{\sqrt{1+2x}-\sqrt{1-2x}}{x}$$

$$14. \lim_{x \rightarrow 7} \frac{x-7}{\sqrt{x+2}-3}$$

Level 3: Sulit (Bentuk Kompleks)

$$15. \lim_{x \rightarrow 4} \frac{3 - \sqrt{x^2 - 7}}{x^2 - 16}$$

$$16. \lim_{x \rightarrow 2} \frac{\sqrt{x+2} - \sqrt{3x-2}}{x-2}$$

$$17. \lim_{x \rightarrow 1} \frac{\sqrt{x^2+3} - 2}{x-1}$$

$$18. \lim_{x \rightarrow 0} \frac{x^2}{\sqrt{x^2+1} - 1}$$

$$19. \lim_{x \rightarrow 3} \frac{\sqrt{2x-2} - 2}{\sqrt{3x} - 3}$$

20. $\lim_{x \rightarrow 2} \frac{4 - x^2}{3 - \sqrt{x^2 + 5}}$