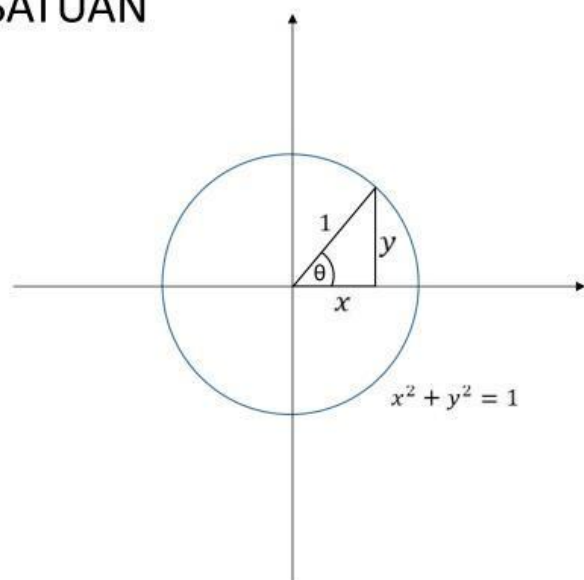


Identitas Trigonometri

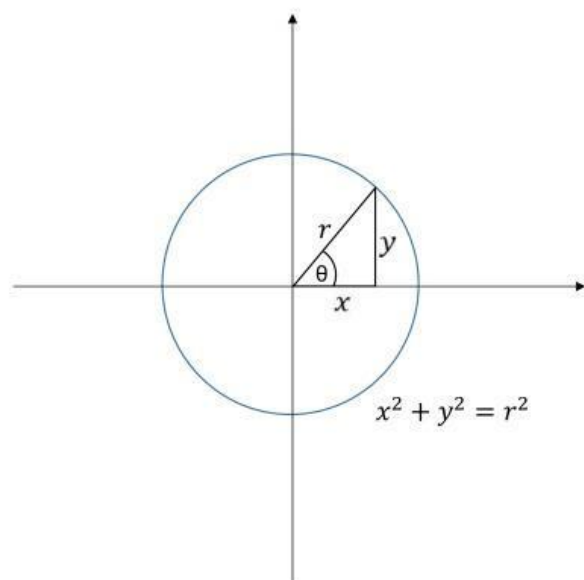
PERBANDINGAN TRIGONOMETRI DENGAN MENGGUNAKAN LINGKARAN SATUAN

- $\sin \theta = \frac{y}{1}$
- $\cos \theta = \frac{x}{1}$
- $\tan \theta = \frac{y}{x}$
- $\sec \theta = \frac{1}{\cos \theta} = \frac{1}{x}$
- $\csc \theta = \frac{1}{\sin \theta} = \frac{1}{y}$
- $\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{x}{y}$



PERBANDINGAN TRIGONOMETRI DENGAN JARI-JARI r

- $\sin \theta = \frac{y}{r}$
- $\cos \theta = \frac{x}{r}$
- $\tan \theta = \frac{y}{x}$
- $\sec \theta = \frac{r}{x}$
- $\csc \theta = \frac{r}{y}$
- $\cot \theta = \frac{x}{y}$



Identitas Kebalikan

$$1. \sin \theta = \frac{1}{\csc \theta} \text{ atau } \csc \theta = \frac{1}{\sin \theta}$$

$$2. \cos \theta = \frac{1}{\sec \theta} \text{ atau } \sec \theta = \frac{1}{\cos \theta}$$

$$3. \tan \theta = \frac{1}{\cot \theta} \text{ atau } \cot \theta = \frac{1}{\tan \theta} \text{ (Silahkan ananda buktikan!!)}$$

Bukti:

$$\begin{aligned} 1. \sin \theta &= \frac{x}{r} & \text{atau} & & \csc \theta &= \frac{r}{x} \\ &= \frac{1}{\frac{r}{x}} & & & &= \frac{1}{\frac{x}{r}} \\ &= \frac{1}{\csc \theta} & & & &= \frac{1}{\sin \theta} \end{aligned}$$

Terbukti bahwa

$$\sin \theta = \frac{1}{\csc \theta}$$

Terbukti bahwa

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\begin{aligned} 2. \cos \theta &= \frac{y}{r} & \text{atau} & & \sec \theta &= \frac{r}{y} \\ &= \frac{1}{\frac{r}{y}} & & & &= \frac{1}{\frac{y}{r}} \\ &= \frac{1}{\sec \theta} & & & &= \frac{1}{\cos \theta} \end{aligned}$$

Terbukti bahwa

$$\cos \theta = \frac{1}{\sec \theta}$$

Terbukti bahwa

$$\sec \theta = \frac{1}{\cos \theta}$$

Identitas Kebalikan

$$1. \sin \theta = \frac{1}{\csc \theta} \text{ atau } \csc \theta = \frac{1}{\sin \theta}$$

$$2. \cos \theta = \frac{1}{\sec \theta} \text{ atau } \sec \theta = \frac{1}{\cos \theta}$$

3. $\tan \theta = \frac{1}{\cot \theta}$ atau $\cot \theta = \frac{1}{\tan \theta}$ (Silahkan ananda buktikan!!)

Bukti:

Cara I:

$$1. \quad \sin \theta = \frac{x}{r} \quad \text{atau} \quad \csc \theta = \frac{r}{x}$$

$$= \frac{1}{\frac{r}{x}} \quad = \frac{1}{\frac{x}{r}}$$

$$= \frac{1}{\csc \theta} \quad = \frac{1}{\sin \theta}$$

Dengan Cara I dan Cara II diperoleh bahwa

$$\sin \theta = \frac{1}{\csc \theta} \text{ dan } \csc \theta = \frac{1}{\sin \theta} \text{ terbukti}$$

Cara II:

$$\frac{1}{\csc \theta} = \frac{1}{\frac{r}{x}} = \frac{x}{r} = \sin \theta$$

atau $\frac{1}{\sin \theta} = \frac{1}{\frac{r}{x}}$
 $= \frac{x}{r}$
 $= \csc \theta$

Identitas Perbandingan

1. $\tan \theta = \frac{\sin \theta}{\cos \theta}$

2. $\cot \theta = \frac{\cos \theta}{\sin \theta}$ (Silahkan ananda buktikan!!)

Bukti :

1. CARA I: $\frac{\sin \theta}{\cos \theta} = \frac{\frac{x}{r}}{\frac{y}{r}} = \frac{x}{y} = \tan \theta$

CARA II: $\tan \theta = \frac{x}{y} = \frac{\frac{x}{r}}{\frac{y}{r}} = \frac{\sin \theta}{\cos \theta}$

Dari cara I dan cara II diperoleh

bahwa $\tan \theta = \frac{\sin \theta}{\cos \theta}$ terbukti

Identitas Pythagoras

Berdasarkan teorema Pythagoras

$$x^2 + y^2 = r^2$$

Kita bagi kedua ruas dengan r^2 , sehingga

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

Diketahui $\sin \theta = \frac{x}{r}$ dan $\cos \theta = \frac{y}{r}$, sehingga

$$(\sin \theta)^2 + (\cos \theta)^2 = 1 \dots (i)$$

Bentuk alternatif:

$$(\sin \theta)^2 = 1 - (\cos \theta)^2$$

$$(\cos \theta)^2 = 1 - (\sin \theta)^2$$

Berdasarkan identitas Pythagoras (i)

$$(\sin \theta)^2 + (\cos \theta)^2 = 1$$

Dapat diperoleh identitas kedua dengan cara membagi kedua ruas dengan $(\sin \theta)^2$

yaitu

$$\frac{(\sin \theta)^2}{(\sin \theta)^2} + \frac{(\cos \theta)^2}{(\sin \theta)^2} = \frac{1}{(\sin \theta)^2}$$

$$1 + \frac{(\cos \theta)^2}{(\sin \theta)^2} = \frac{1}{(\sin \theta)^2}$$

Diketahui $\cot \theta = \frac{\cos \theta}{\sin \theta}$ dan $\csc \theta = \frac{1}{\sin \theta}$, sehingga

$$1 + (\cot \theta)^2 = (\csc \theta)^2 \dots (ii)$$

Bentuk alternatif:

$$(\cot \theta)^2 = (\csc \theta)^2 - 1$$

$$1 = (\csc \theta)^2 - (\cot \theta)^2$$

Tugas

1. Buktikan identitas kebalikan berikut

a. $\tan \theta = \frac{1}{\cot \theta}$

b. $\cot \theta = \frac{1}{\tan \theta}$

2. Buktikan identitas perbandingan dari $\cot \theta = \frac{\cos \theta}{\sin \theta}$

3. Sederhanakanlah

a. $-\frac{\csc \theta}{\cot \theta}$

b. $(1 - (\sin \theta)^2)(1 + (\cot \theta)^2)$

4. Buktikan identitas berikut $\sin \theta \sec \theta = \tan \theta$