



رياضيات 1

Mathematics 1 The Functions

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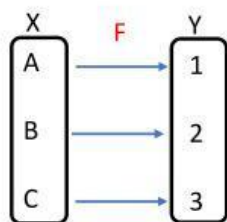
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*** Defintion :**

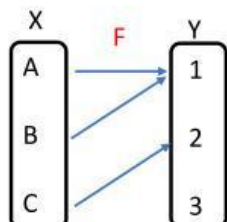
A functions is a relation between the elements of two unempty sets ,where every element in a set has only one image in the other set.

this can be written in the form $y = f(x)$ or $f : X \rightarrow Y$

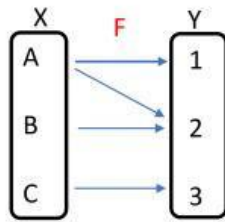
x is independent variable, y is dependen variable.



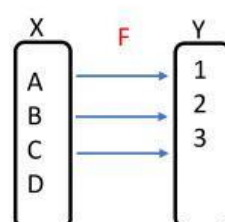
Function



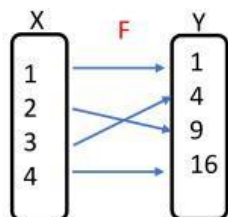
Function



Not Function



Not Function



$$f : X \rightarrow Y \quad , \quad f(x) = y$$

$1 \rightarrow 1$	$f(1) = 1$
$2 \rightarrow 4$	$f(2) = 4$
$3 \rightarrow 9$	$f(3) = 9$
$4 \rightarrow 16$	$f(4) = 16$

***Domain of the function:**

where X is the domain of the function f and may be denoted by D_f .

Y is the Co-domain of f .

***Range of the function:**

the range is a set of all image of the elements of the domain and it is denoted by R_f .

we note that $R_f \subset Y$

***Remarks to Find the Nutral Domain:**

1. For rational functions the domain is $\mathbb{R}$ except any number make the dominator to be zero i.e.

domain = $\mathbb{R} - \{\text{zeros of dominator}\}$.

2. When the function contain square root or any even root $\sqrt[4]{}$, $\sqrt[6]{}$ hence the domain will be $\mathbb{R}$ except the numbers which make the value under the root sign to be negative value.

Example : Find the domain and range of the functions

1) $f(x) = \frac{2x}{x-3}$

R- {zeros of dominator}

$x-3=0 \rightarrow x=3 \rightarrow D_f = R-\{3\}$

$y = \frac{2x}{x-3} \rightarrow yx-3y=2x \rightarrow yx-2x=3y \rightarrow x(y-2)=3y \rightarrow x = \frac{3y}{y-2}$

$R_f = R-\{2\}$

2) $f(x) = \sqrt{x-1}$

the value under the root ≥ 0

$x-1 \geq 0 \rightarrow x \geq 1 \rightarrow D_f = [1, \infty[\text{ or } [1, \infty)$

$R_f = [0, \infty[$

3) $y = \frac{1}{\sqrt{x^2-1}}$

the value under the root > 0

$x^2-1 > 0 \rightarrow x^2 > 1 \rightarrow \pm x > 1$

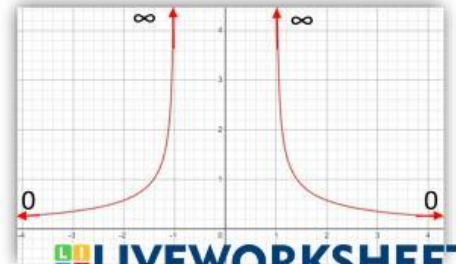
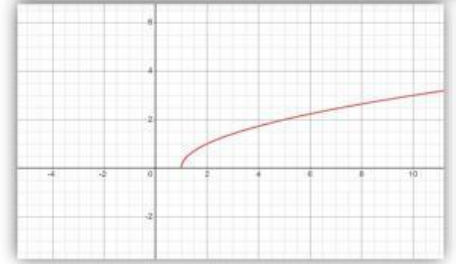
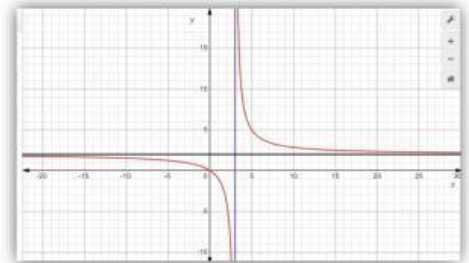
$x > 1 \text{ or } -x > 1$

$x > 1 \text{ or } x < -1$

$D_f = R - [-1, 1]$

at $x = \pm 1 \rightarrow y = \infty$, $x = \pm \infty \rightarrow y = 0$

$R_f =]0, \infty[$



$$4) f(x) = \sqrt{4 - \sqrt{x}}$$

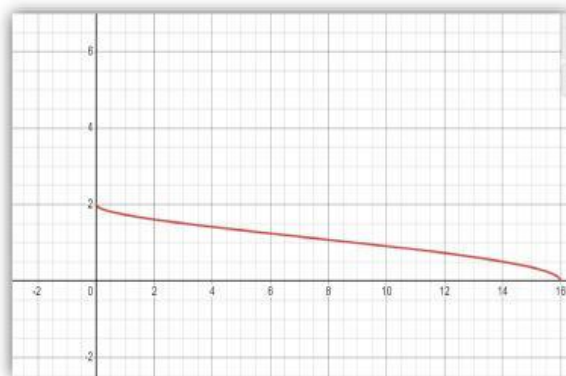
$$\sqrt{x} \rightarrow \boxed{x \geq 0}$$

$$\sqrt{4 - \sqrt{x}} \rightarrow 4 - \sqrt{x} \geq 0 \rightarrow 4 \geq \sqrt{x} \rightarrow \boxed{16 \geq x}$$

$$0 \leq x \leq 16, \quad D_f = [0, 16]$$

$$\text{at } x=0 \rightarrow y=2, \quad x=16 \rightarrow y=0$$

$$R_f = [0, 2]$$



$$5) y = \sqrt{4 - x^2}$$

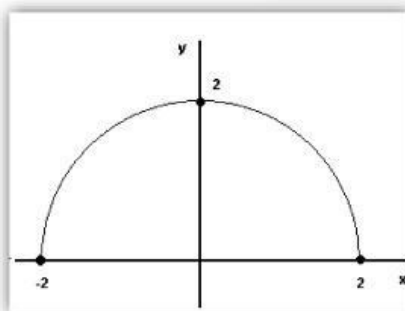
$$4 - x^2 \geq 0 \rightarrow 4 \geq x^2 \rightarrow 2 \geq \pm x$$

$$2 \geq x \quad \text{or} \quad 2 \geq -x$$

$$-2 \leq x$$

$$-2 \leq x \leq 2 \Rightarrow D_f = [-2, 2]$$

$$R_f = [0, 2]$$



$$*) y = \sqrt{4 - x^2} \Rightarrow y \text{ is always positive}$$

$$y^2 = 4 - x^2 \rightarrow x^2 + y^2 = 4$$

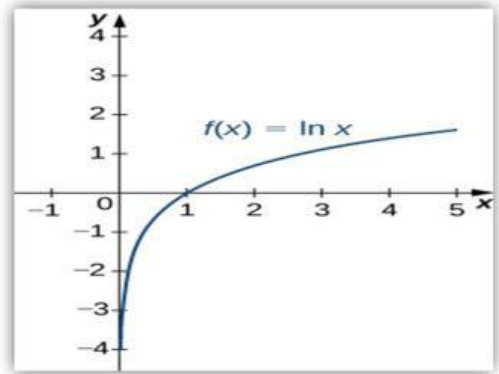
$$\boxed{x^2 + y^2 = a^2}$$

*) $y = \ln x$

the value inside $\ln > 0$

$$x > 0 \rightarrow D_f =]0, \infty[$$

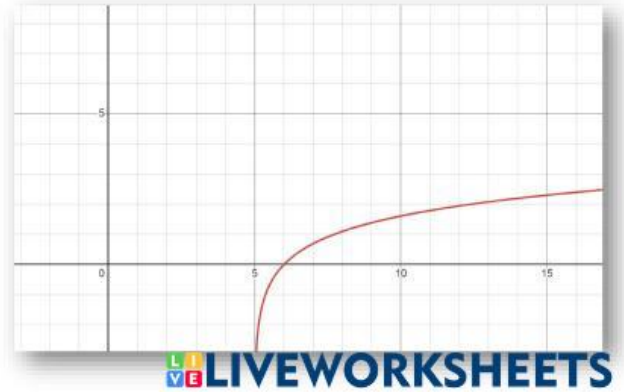
$$R_f =]-\infty, \infty[= \mathbb{R}$$



4) $y = \ln(x - 5)$

$$x - 5 > 0 \rightarrow x > 5 \rightarrow D_f =]5, \infty[$$

$$R_f =]-\infty, \infty[= \mathbb{R}$$



Example: Find the range of the functions

$$1) y = \frac{1}{2 - \sin x}$$

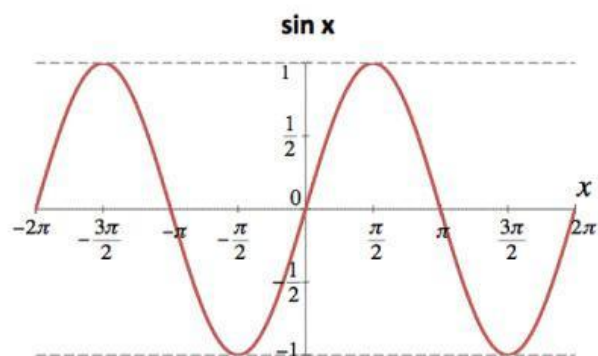
$$2y - y \sin x = 1 \rightarrow y \sin x = 2y - 1 \rightarrow \sin x = \frac{2y - 1}{y}$$

$$\sin x = 2 - \frac{1}{y}$$

$$\boxed{-1 \leq \sin x \leq 1}$$

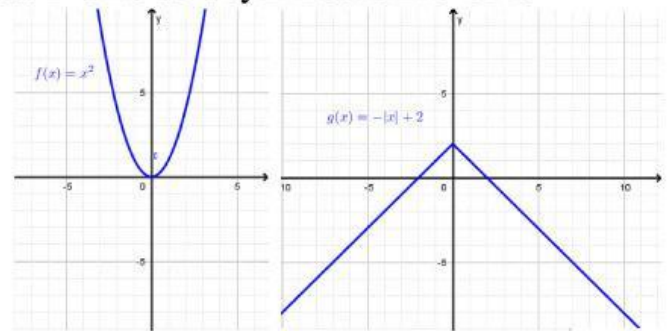
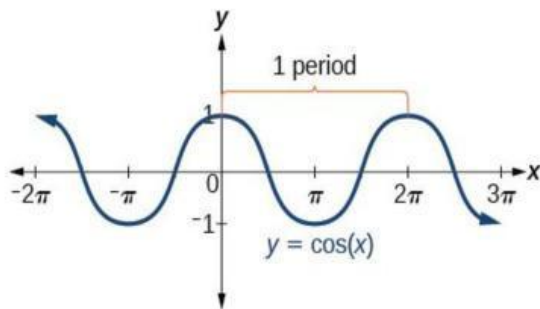
$$-1 \leq 2 - \frac{1}{y} \leq 1 \rightarrow -3 \leq \frac{-1}{y} \leq -1 \rightarrow 3 \geq \frac{1}{y} \geq 1$$

$$\frac{1}{3} \leq y \leq 1 \Rightarrow R_f = \left[\frac{1}{3}, 1\right]$$

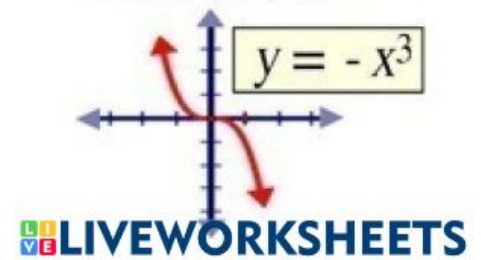
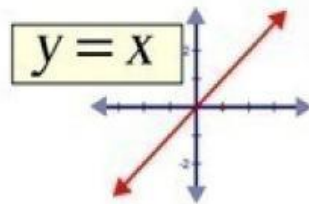
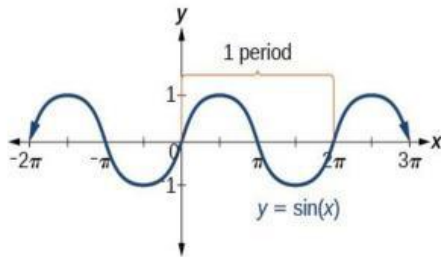


Even and Odd Functions

$f(x)$ is an **even** function if $f(-x) = f(x)$ and has Y-axis as a symmetric line of it



$f(x)$ is an **odd** function if $f(-x) = -f(x)$ and the curve of $f(x)$ is symmetric about the origin



Ex : Check the following functions even, odd, or not : -

1) $f(x) = x^2 - 1$

$$f(-x) = (-x)^2 - 1 = x^2 - 1 = f(x)$$

$\therefore f(x)$ is even function

2) $f(x) = x^2 + x$

$$f(-x) = (-x)^2 + (-x) = x^2 - x \neq f(x)$$

$$\text{and } f(-x) \neq -f(x)$$

$\therefore f(x)$ neither odd nor even function

3) $f(x) = \frac{x^3 + 2x}{x^4 - 3x^2 + 6}$

$$f(-x) = \frac{(-x)^3 + 2(-x)}{(-x)^4 - 3(-x)^2 + 6} = \frac{-(x^3 + 2x)}{x^4 - 3x^2 + 6} = -f(x)$$

$$\rightarrow f(-x) = -f(x)$$

$\therefore f(x)$ is odd function

Types of the function

1. One to one function (injective)

$$\boxed{\forall x \in D_f, x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)} \Rightarrow \therefore f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

Ex: $f(x) = x^2$

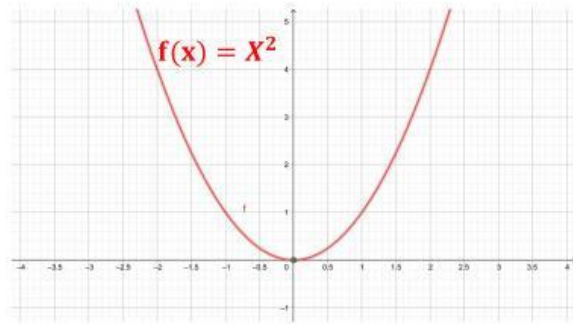
let $f(x_1) = f(x_2)$

$$x_1^2 = x_2^2 \Rightarrow x_1^2 - x_2^2 = 0$$

$$(x_1 - x_2)(x_1 + x_2) = 0$$

$$x_1 = x_2 \quad \text{or} \quad x_1 = -x_2$$

$f(x)$ is not one to one function

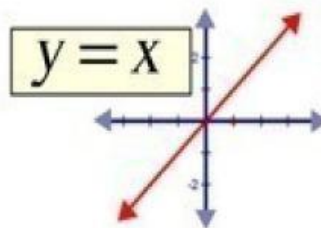


Ex: $f(x) = x$

let $f(x_1) = f(x_2)$

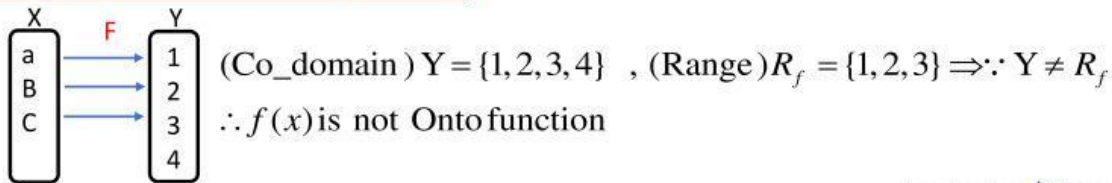
$$x_1 = x_2$$

$f(x)$ is one to one function



2. Onto function (surjective)

$$\text{Range} = \text{Co_domain} \quad (R_f = Y)$$



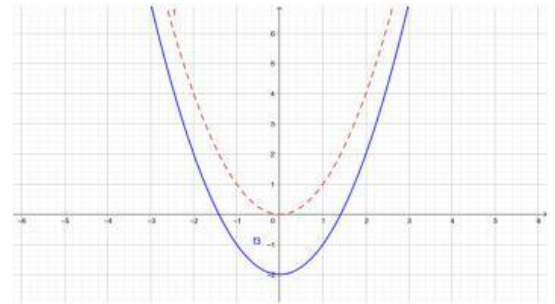
Ex: Is $f(x) = x^2 - 2$ Onto where $f: \mathbb{R} \rightarrow \mathbb{R}$?

($f: \text{domain} \rightarrow \text{Co_domain}$)

$R_f = [-2, \infty[$, $Y = \mathbb{R} \rightarrow R_f \neq Y \rightarrow \therefore f(x)$ is not onto function

Ex: Is $f(x) = x^2 - 2$ Onto where $f: \mathbb{R} \rightarrow [-2, \infty[$?

$R_f = [-2, \infty[$, $Y = [-2, \infty[\rightarrow R_f = Y \rightarrow \therefore f(x)$ is onto function



3. Bijective function

if it is one to one (injective) and onto (surjective)

*Invers of a Function:

A function $f(x)$ has an invers function $f^{-1}(x)$ if and only if it is bijective (or at least is one to one)

Note that :

1. $f^{-1}(x) \neq \frac{1}{f(x)}$

2. The domain of $f^{-1}(x)$ is the co_domain of $f(x)$ and its range is the domain of f

3. The function and its invers are symmetric at $y = x$

Ex: Show that $f(x) = 2x + 1$ has invers and find it

$$D_f = \mathbb{R} \quad , \quad R_f = \mathbb{R}$$

$$\text{let } f(x_1) = f(x_2)$$

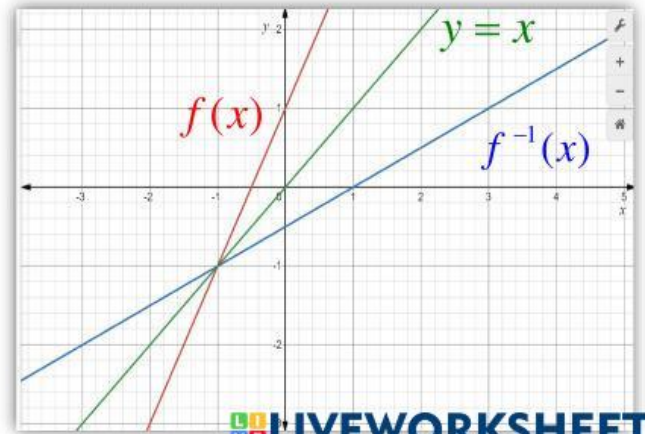
$$2x_1 + 1 = 2x_2 + 1 \Rightarrow 2x_1 - 2x_2 = 0 \Rightarrow 2(x_1 - x_2) = 0$$

$$\therefore x_1 = x_2 \Rightarrow f(x) \text{ is one to one function} \Rightarrow f(x) \text{ has invers}$$

$$\boxed{f(x) = y = 2x + 1} \Rightarrow x = \frac{y - 1}{2}$$

$$f^{-1}(y) = \frac{y - 1}{2}$$

$$\boxed{f^{-1}(x) = \frac{x - 1}{2}}$$



Let f, g two functions

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$

$\therefore g$ is the inverse of f and f is the inverse of g

$$f(x) = 2x + 1, \quad f^{-1}(x) = \frac{x-1}{2} = g(x)$$

$$f(g(x)) = 2\left(\frac{x-1}{2}\right) + 1 = x \quad \text{and} \quad g(f(x)) = \frac{(2x+1)-1}{2} = x$$

$\therefore g$ is the inverse of f and f is the inverse of g

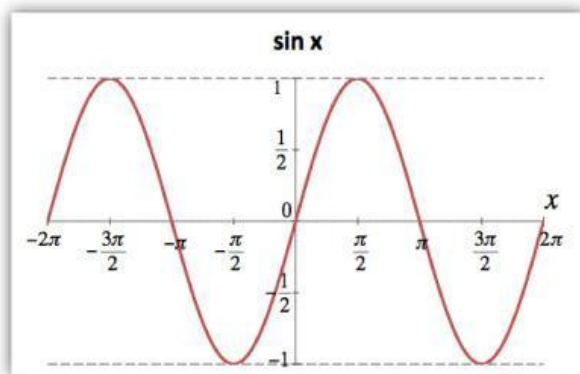
Properties of Function

* Periodic Function

$$f(x \pm p) = f(x)$$

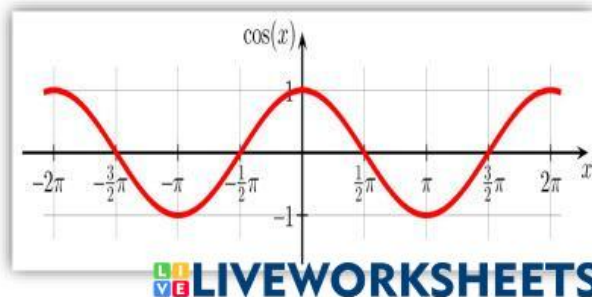
Ex: $f(x) = \sin(x)$

$$p = 2\pi \rightarrow \sin(x \pm 2\pi) = \sin(x)$$



Ex: $f(x) = \cos(x)$

$$p = 2\pi \rightarrow \cos(x \pm 2\pi) = \cos(x)$$



Thank you