English Spanish

3.7 Direct Variation

STATE STANDARDS
MA.7.A.1.4
MA.7.A.1.5

Essential Question How can you use a graph to show the relationship between two variables that vary directly? How can you use an equation?

1 ACTIVITY: Math in Literature

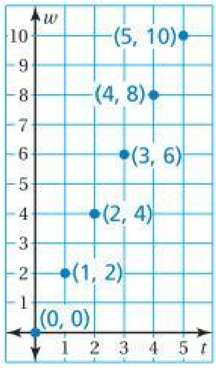
Gulliver's Travels was written by Jonathan Swift and published in 1725. Gulliver was shipwrecked on the island Lilliput, where the people were only 6 inches tall. When the Lilliputians decided to make a shirt for Gulliver, a Lilliputian tailor stated that he could determine Gulliver's measurements by simply measuring the distance around Gulliver's thumb. He said "Twice around the thumb equals once around the wrist. Twice around the wrist is once around the neck. Twice around the neck is once around the waist."

Work with a partner. Use the tailor's statement to complete the table.

Thumb, t	Wrist, w	Neck, n	Waist, x
0 in.	0 in.		
1 in.	2 in.		
2 in.	4 in.		
3 in.	6 in.		
4 in.	8 in.		
5 in.	10 in.		

2 EXAMPLE: Drawing a Graph

Use the information from Activity 1 to draw a graph of the relationship between the distance around the thumb *t* and the distance around the wrist *w*.



Use the table to write ordered pairs. Then plot the ordered pairs.

(0, 0), (1, 2), (2, 4), (3, 6), (4, 8), (5, 10)

Notice the following about the graph:

1. All the points lie on a line.
2. The line passes through the origin.

This type of relationship is called **direct variation**. You can write an equation to describe the relationship between t and w.

$w = 2t$ Wrist is twice thumb.

3 ACTIVITY: Drawing a Graph

Work with a partner. Use the information from Activity 1 to draw a graph of the relationship. Write an equation that describes the relationship between the two variables.

a. Thumb t and neck n ($n =$ ___ t)

b. Wrist w and waist x ($x =$ ___ w)

c. Wrist w and thumb t ($t =$ ___ w)

d. Waist x and wrist w ($w =$ ___ x)

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you use a graph to show the relationship between two variables that vary directly? How can you use an equation?

5. Give a real-life example of two variables that vary directly.

6. Work with a partner. Use string to find the distance around your thumb, wrist, and neck. Do your measurements agree with those of the tailor in *Gulliver's Travels*? Explain your reasoning.

Practice Use what you learned about direct variation to complete Exercises 4–7 on page 140.

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3.7 Lesson

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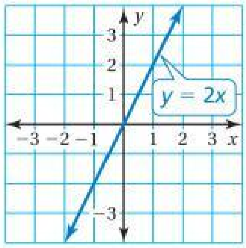
Key Vocabulary
direct variation, p. 138

Key Idea

Direct Variation

Words Two quantities x and y show **direct variation** when $y = kx$, where k is a number and $k \neq 0$.

Graph The graph of $y = kx$ is a line that passes through the origin.



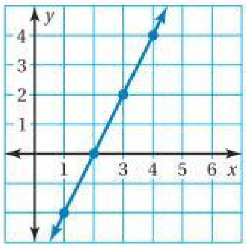
EXAMPLE 1 Identifying Direct Variation

Tell whether x and y show direct variation. Explain your reasoning.

a.

x	1	2	3	4
y	−2	0	2	4

Plot the points. Draw a line through the points.

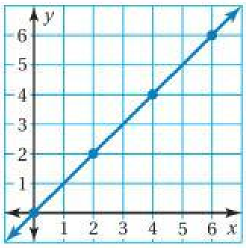


The line does not pass through the origin. So, x and y do *not* show direct variation.

b.

x	0	2	4	6
y	0	2	4	6

Plot the points. Draw a line through the points.



The line passes through the origin. So, x and y show direct variation.

Study Tip
Other ways to say that x and y show direct variation are "y varies directly with x" and "x and y are directly proportional."

EXAMPLE 2 Identifying Direct Variation

Tell whether x and y show direct variation. Explain your reasoning.

a. $y + 1 = 2x$

$y = 2x - 1$ Solve for y.

The equation *cannot* be written as $y = kx$. So, x and y do *not* show direct variation.

b. $\frac{1}{2}y = x$

$y = 2x$ Solve for y.

The equation can be written as $y = kx$. So, x and y show direct variation.

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On Your Own

Now You're Ready
Exercises 8–21

Tell whether x and y show direct variation. Explain your reasoning.

1.

x	y
0	−2
1	1
2	4
3	7

2.

x	y
1	4
2	8
3	12
4	16

3.

x	y
−2	4
−1	2
0	0
1	2

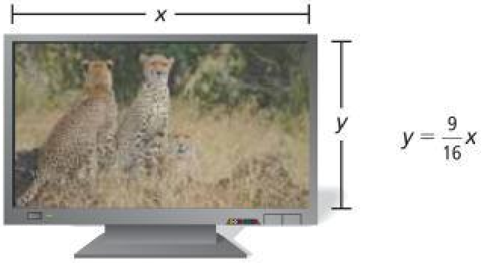
4. $xy = 3$

5. $x = \frac{1}{3}y$

6. $y + 1 = x$

EXAMPLE 3 Using a Direct Variation Model

The height y of a television screen varies directly with its width x.



a. Find the height when the width is 48 inches.

b. Sketch the graph of the equation.

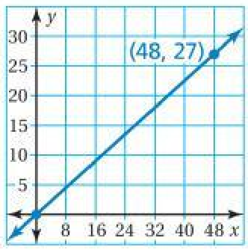
a. Use the equation to find the height when $x = 48$ inches.

$y = \frac{9}{16}(48)$ Substitute 48 for x.

$= 27$ Simplify.

So, when the width is 48 inches, the height is 27 inches.

b. To sketch a graph, plot the point (48, 27). Then draw the line that passes through this point and the origin.



On Your Own

Now You're Ready
Exercises 24–29

7. Your earnings y (in dollars) vary directly with the number x of lawns you mow. Use the equation $y = 7.5x$ to find how much you earn when you mow 5 lawns.

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3.7 Exercises

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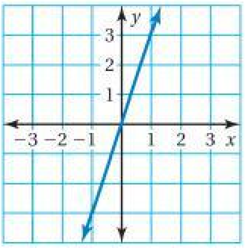
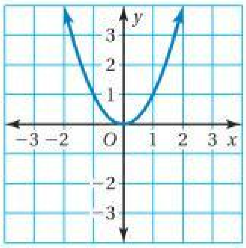
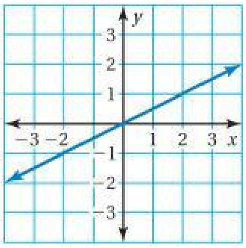
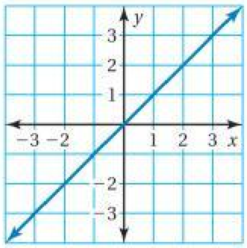
Vocabulary and Concept Check

1. **VOCABULARY** What does it mean for x and y to vary directly?
2. **WRITING** What point is on the graph of every direct variation equation?
3. **WHICH ONE DOESN'T BELONG?** Which graph does *not* belong with the other three? Explain your reasoning.

Practice and Problem Solving

Tell whether x and y show direct variation. Explain your reasoning.

4. $(-1, -1), (0, 0), (1, 1), (2, 2)$
5. $(-4, -2), (-2, 0), (0, 2), (2, 4)$
6. $(1, 2), (1, 4), (1, 6), (1, 8)$
7. $(2, 1), (6, 3), (10, 5) (14, 7)$

(1) 8.

x	1	2	3	4
y	2	4	6	8

9.

x	−2	−1	0	1
y	0	2	4	6

10.

x	−1	0	1	2
y	−2	−1	0	1

11.

x	4	8	12	16
y	1	2	3	4

12.

x	−1	0	1	2
y	1	0	1	2

13.

x	3	6	9	12
y	2	4	6	8

(2) 14. $y - x = 4$

15. $x = \frac{2}{5}y$

16. $y + 3 = x + 6$

17. $y - 5 = 2x$

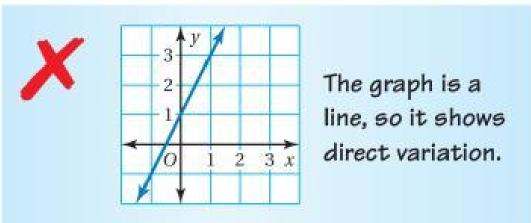
18. $x - y = 0$

19. $\frac{x}{y} = 2$

20. $8 = xy$

21. $x^2 = y$

22. **ERROR ANALYSIS** Describe and correct the error in telling whether x and y show direct variation.



23. **RECYCLING** The table shows the profit y for recycling x pounds of aluminum. Tell whether x and y show direct variation.

Aluminum, x	10	20	30	40
Profit, y	\$4.50	\$9.00	\$13.50	\$18.00

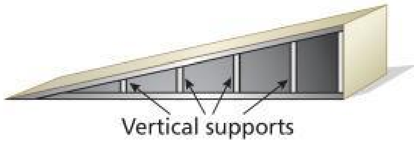
The variables x and y vary directly. Use the values to write an equation that relates x and y.

3 **24.** $y = 4; x = 2$ **25.** $y = 25; x = 5$ **26.** $y = 60; x = 15$

27. $y = 72; x = 3$ **28.** $y = 20; x = 12$ **29.** $y = 45; x = 40$

30. MEASUREMENT Write a direct variation equation that relates x inches to y centimeters.

31. JET SKI RAMP Design a jet ski ramp. Show how you can use direct variation to plan the heights of the vertical supports.



32. JUPITER The weight of an object in our solar system varies directly with the weight of the object on Earth.

a. Copy and complete the table.

b. RESEARCH Why does weight vary throughout our solar system?

Location	Earth	Jupiter	Moon
Weight (lb)	100	214	
Weight (lb)	120		20

Minutes, x	500	700	900	1200
Cost, y	\$40	\$50	\$60	\$75

33. CELL PHONE PLANS Tell whether x and y show direct variation. If so, write an equation of direct variation.

34. CHLORINE The amount of chlorine in a swimming pool varies directly with the volume of water. The pool has 2.5 milligrams of chlorine per liter of water. How much chlorine is in the pool?

35. Critical Thinking Is the graph of every direct variation equation a line? Does the graph of every line represent a direct variation equation? Explain your reasoning.

Fair Game Review What you learned in previous grades & lessons

Solve the equation. SECTION 2.5

36. $-4x = 36$ **37.** $\frac{y}{6} = -10$ **38.** $-\frac{3}{4}m = 24$ **39.** $-17 = \frac{2}{7}d$

40. MULTIPLE CHOICE Which rate is *not* equivalent to 180 feet per 8 seconds? SECTION 3.1

(A) $\frac{225 \text{ ft}}{10 \text{ sec}}$ (B) $\frac{45 \text{ ft}}{2 \text{ sec}}$ (C) $\frac{135 \text{ ft}}{6 \text{ sec}}$ (D) $\frac{180 \text{ ft}}{1 \text{ sec}}$