

Definition of regular singular points

Consider a ordinary differential equation

$$P(x)y'' + Q(x)y' + R(x)y = 0.$$

If

i) $\lim_{x \rightarrow x_0} (x - x_0) \frac{Q(x)}{P(x)}$

ii) $\lim_{x \rightarrow x_0} (x - x_0)^2 \frac{R(x)}{P(x)}$

both are exist, then is a point.

first-order

second-order

$x = x_0$

$x = x_n$

ordinary

regular singular

singular